Standing Waves: The Simple Guitar

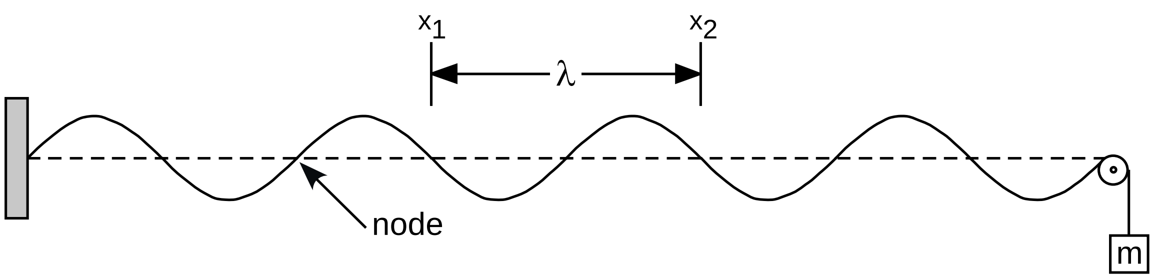
Fall 2007

Introduction

In this experiment you will examine the relationship between tension, wavelength and velocity of waves on a string. You will also calculate the *linear density*, μ, of the string.

Theory

Waves on a string clamped at both ends appear to stand still if half their *wavelength* just fits the length of the string between the clamps.



The following quantities can be measured directly: The *wavelength*, λ (the distance between ***three*** consecutive 'nodes'); the *frequency*, ***f*** of the wave (for this experiment, f = 60 Hz); and the *string tension*, **T**.

From the frequency and wavelength you can calculate the *wave velocity*, $v = f \cdot \lambda$. Applying Newton's law to a string gives a theoretical connection between wave velocity, the tension on the string, and the *linear density*, μ:

$$v = \sqrt{\frac{T}{\mu}}$$

This expression can be written as $v = \frac{1}{\sqrt{\mu}}\sqrt{T} = c\sqrt{T}$, where $c = \frac{1}{\sqrt{\mu}}$. The units for μ are *mass* per *length*. You will analyze your data using KaleidaGraph to calculate the value of μ for your string.

Experiment

1. Create a data table with the following headers:

m (kg)	x_1 (m)	x_2 (m)	$\lambda = \lvert x_2 - x_1 \rvert$ (m)	$v = f \cdot \lambda$ (m/s)	T = mg (N)

2. Suspend 500 *g* from the string (*remember that the mass of the hanger is 50 g!*), and turn on the string vibrator.
3. Measure the wavelength λ (the distance between three consecutive nodes). Use this measurement to determine the wave velocity. Choose nodes near the center of the string; don't use the nodes that appear at both ends of the string. Calculate the string tension **T** in the string, using g = 9.80 m/s^2.
4. Begin a hand-drawn graph of wave velocity as a function of string tension, and plot your first point. *Remember to plot the points as you collect your data!*
5. Repeat the measurements of λ, v, and **T** for other tensions down to as small a value as possible.

6. A simple method of calculating the linear density would be to determine the mass of your string, and measure its length; dividing the two would give you μ. It is difficult to disassemble the apparatus, so you will determine the mass of an identical 'sample' string and measure its length to get a static calculation of μ.

Analysis

The relationship between wave velocity and string tension follows a power law: $y = ax^n$, where the power $\boldsymbol{n}$ is 0.5 (because of the square root). We face the same dilemma as with the "Simple Pendulum" experiment; Excel will attempt to adjust $\boldsymbol{n}$ as well as $\boldsymbol{a}$ to get the best possible fit; unfortunately, anything other than $n = 0.5$ produces lousy results. So, to improve our analysis, we will again use KaleidaGraph.

7. Begin creating your table of data in KaleidaGraph by first labeling the data columns (by default, they're labeled A, B, C, etc.). Double-click the column title and change columns A and B to T and v (be sure to include units!!). Then enter your T and v data (***Note:*** *Do not include the coordinate {0,0} in your table!*).
8. Now you will create a graph of v vs. T as follows:
   i. From the **Gallery** menu, choose **Linear**, then **Scatter**. In the "Plot" dialog that appears, click the radio buttons next to L and T to graph them on the X and Y-axis, respectively (*now you see why it's important to label the columns!*). Click the **New Plot** button.
   ii. Select the **Curve Fit** menu; you'll see many types available. Choose **General**, then **Square Root** (*Note:* this is a custom fit added by your instructor). Click the box next to $\langle \boldsymbol{T} \rangle$, then click **OK**. A curve will be fit to your data, and a table of results will appear. Click and drag the table to move it on your graph so that it doesn't obscure the curve (*Note:* if the table of data does not appear, click the **Plot** menu, and select **Display Equation**).
   iii. Before printing your graph, you should fix it up a bit.
      a. Turn off the legend by choosing **Plot**, **Display Legend**.
      b. Create a proper graph title (it probably says "Data 1" now); double-click the existing title and edit it. Press Enter, type your names and click **OK**.
      c. Extend the graph axes so that they include the origin: click the **Plot** menu and choose **Axis Options**. In the dialog box that appears, choose the **X** or **Y** tab, then change the **Minimum** value to 0. Click **OK** when finished.
      d. Choose **Print Graphics** from the **File** menu to print the graph (*Don't use the print button on the toolbar – it might print too many copies!*).
9. Use your measured value of the coefficient a to calculate an experimental value for the mass per unit length of the string, μ.

Discussion

- Report your two values for μ. Do they agree with each other? Discuss what may have contributed to the difference between values.