Vector Addition

Fall 2007

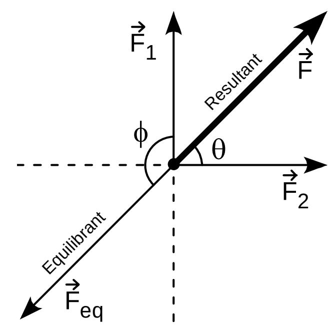
Introduction

In this experiment you will use a *force table* to learn how vectors are used to represent forces, and practice adding vectors graphically and algebraically.

Theory

Forces are vector quantities, described by both their magnitude and direction. We will use masses suspended from strings to exert force on an object – a metal ring. The *resultant* is a single force calculated as the vector sum of the forces exerted on the object. We will be able to calculate the magnitude and direction of the resultant vector on the force table by examining the *equilibrant*, which is a single force that establishes equilibrium by balancing two or more forces. *So, the equilibrant has the same magnitude, but opposite direction as the resultant.*

Experiment 1: Addition of Two Vectors



You will add two vectors $\left(\vec{F}_1, \vec{F}_2\right)$ and calculate the magnitude and direction of the resultant $\left(\vec{F}\right)$ and equilibrant $\left(\vec{F}_{eq}\right)$, both algebraically and graphically. The results will also be measured on the force table.

Measured:

1. You will set up the two vectors to be added on the force table. Position the table in the desired position on the lab bench and adjust the top to be level according to the bubble level.
2. You will use *three* pulleys for this portion of the experiment; if necessary, remove the fourth pulley and set it aside. Attach two pulleys to represent the forces to be added, $\vec{F}_1$ and $\vec{F}_2$; their direction is indicated in the figure above. The third pulley will represent the *equilibrant*, used to balance the other two forces, so its position can be set approximately for now. *Do not over tighten the pulley clamps!*
3. Place the ring with *three* strings over the center post, and pass each string over a pulley. The table below gives the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ (note that the units of "force" have been left in *grams*; this will simplify your calculations and measurements). Attach mass hangars to each string, and place additional mass on the two strings that will represent the magnitudes of $\vec{F}_1$ and $\vec{F}_2$ (note that the hangars have a mass of 50 *g*).

Forces		Measured on Table		Calculated			Measured on Graph	
F_1 (g)	F_2 (g)	F (g)	ϕ°	F (g)	θ°	ϕ°	F (g)	θ°
150	100							

4. Estimate the angle ϕ as follows: Grab the third string (representing the equilibrant), and gently pull the string while moving it left and right with respect to the table. Do so until the ring is centered on the post. Set the third pulley to the position you determined, and hang the string representing the equilibrant over.
5. Now place additional mass on the third string representing the equilibrant. Recall that this force will balance the resultant force of $\vec{F}_1$ and $\vec{F}_2$. Adjust ϕ and the amount of force as necessary. Make sure the strings pass straight over each pulley.

6. The system is balanced (in a state of *static equilibrium*) when the ring is centered on the central post. Record your measurements when the system is balanced.

Calculated:

7. Use trigonometry to calculate the magnitude and direction, θ, of the resultant vector, $\vec{F}$. Also calculate the angle $\phi = \theta + 90°$, which will be the angle of the equilibrant as measured on the force table (make sure you understand *why* you add 90° to θ to get ϕ).

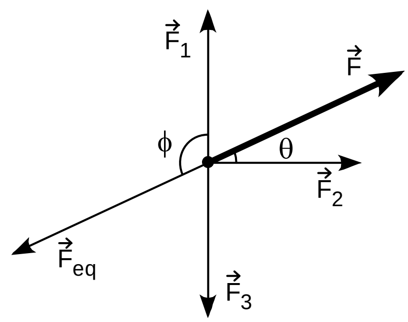
Graphical:

8. You will now add the vectors graphically. Pick a point near the center of a sheet of graph paper as the origin (use the intersection of two darker lines). Using a scale of 0.5 *cm* (1 block) = 10 *g*, draw $\vec{F}_2$ in its direction and scaled length, beginning at the origin. Now draw $\vec{F}_1$, scaled to the correct magnitude and direction, starting at the head of $\vec{F}_2$.

9. Draw the resultant vector, $\vec{F}$. Measure its magnitude with a ruler (in *cm*), convert back to units of grams and enter in the table. Measure its angle, θ, with a protractor.

10. Again use trigonometry to resolve $\vec{F}_{eq}$ into its x- and y-components, and *lightly* draw and label these components on the graph. You should find that these components have the same magnitude (but opposite direction) from the original vectors, $\vec{F}_1$ and $\vec{F}_2$.

Experiment 2: Addition of Three Vectors



You will now calculate the resultant vector of *three* forces by adding their components, algebraically and graphically, and by using the force table.

1. The table below gives the magnitude and direction for three forces:

Force	Magnitude (*g*)	Direction (°)	F_x (*g*)	F_y (*g*)
$\vec{F}_1$	110	0		
$\vec{F}_2$	90	90		
$\vec{F}_3$	80	180		
		$\sum \vec{F} \Rightarrow$		

2. Remove the ring with three strings from the force table, and put the ring with four strings in its place. Move the three pulleys on the table to the configuration given above, then attach a *fourth* pulley, which will represent the equilibrant.

3. Attach mass hangars and appropriate mass to the three hangars representing the forces to be added. Follow the same procedure as in Experiment 1 to determine the angle and magnitude of the equilibrant. Record these values in your report.

4. Fill in the columns for F_x and F_y, the magnitude of the x- and y-components of each force (look at the sketch – the components are easy to determine!). The sum of these columns gives the x- and y-components of the resultant vector.

5. Use trig to calculate the magnitude and direction of the resultant, $\vec{F}$ from the components you just calculated. Calculate the direction of the equilibrant, ϕ as well.
6. Draw a graph with the same scale as before, showing the vector addition of these three forces. Draw the resultant and equilibrant vectors, and measure their magnitude and angle.

Discussion

- There are far too many numerical results in this experiment to warrant restating them all. Instead, write a short paragraph about what you learned about the interaction of forces from this experiment.
- Compare your results with those predicted by Newton's 2^{nd} law $\left(\sum \vec{F} = ma\right)$.
- Discuss possible sources of error. What assumptions are made about the force tables?