Measuring the Hall weighting function for square and cloverleaf geometries

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We have directly measured the Hall weighting function—the sensitivity of a four-wire Hall measurement to the position of macroscopic inhomogeneities in Hall angle—for both a square shaped and a cloverleaf specimen. Comparison with the measured resistivity weighting function for a square geometry [D. W. Koon and W. K. Chan, Rev. Sci. Instrum. 69, 12 (1998)] proves that the two measurements sample the same specimen differently. For Hall measurements on both a square and a cloverleaf, the function is nonnegative with its maximum in the center and its minimum of zero at the edges of the square. Converting a square into a cloverleaf is shown to dramatically focus the measurement process onto a much smaller portion of the specimen. While our results agree qualitatively with theory, details are washed out, owing to the finite size of the magnetic probe used.

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Resistive and Hall measurements performed on specimens containing macroscopic inhomogeneities—defects much larger than the mean free path—can be considered weighted averages of the local transport properties. One can calculate the relative influence of each portion of the specimen in determining the averaged value for the whole specimen—the "weighting" or "response function"—by noting that the effect of a macroscopic point inhomogeneity on the measured resistivity (Hall angle) is equivalent to the effect produced by locating a point dipole at that position, proportional to and parallel to the local electric field. This weighting function has been calculated for a variety of geometries for both resistivity\textsuperscript{1,3} and Hall measurements\textsuperscript{2-10} and has been directly measured for some resistive\textsuperscript{11} and Hall\textsuperscript{5,9,10} geometries. The reason for the greater interest in the Hall problem is that inhomogeneities of the Hall angle in a specimen may be due to variations either in the material being studied or in the magnetic field itself: most studies have assumed that the field was nonuniform and were studies of how a Hall sensor averages the local field.

Resistive and Hall weighting functions calculated for the same geometry have been shown to differ substantially.\textsuperscript{8} We can thus say that these measurements probe different parts of the same object. We have sought to test this experimentally by investigating the same square geometry for which one of us had previously measured the resistive weighting function.\textsuperscript{11}

We evaporated a layer of bismuth having a sheet resistance of 3\(\Omega\) on a plastic substrate and scribed a 22 mm square in the layer to form our Hall specimen. Probes were attached to three corners using silver paint, with an additional two probes straddling the final corner, which were connected to a potentiometer for nulling the zero-field voltage. A 2 mA, 93 Hz current was sent across opposite corners of the specimen, and a Hall voltage was measured using a lock-in amplifier.

The Hall voltage was the result of the field provided by a magnetic probe consisting of a 5 mm diameter rare earth magnet at the end of a 10 cm nail positioned 3 mm above the specimen. The specimen was mounted on a translation stage which allowed us to change the position of the probe in 5.5 mm steps, for a 5 \(\times\) 5 measurement grid. The voltage perpendicular to the current with both the magnet in place and removed was measured, yielding Hall voltages of up to 13 \(\mu\)V. We have plotted the change in the Hall voltage as a function of position in Fig. 1(a) a quantity directly proportional to the Hall weighting function. Figure 1(b) shows the Hall weighting function as calculated previously\textsuperscript{8} for this geometry.

To test whether cloverleaves focus the measurement process onto a smaller region of the specimen, we inscribed slits \(1/3\) as long as the specimen width, from the midpoint of each edge to form a cloverleaf. Measurements were repeated as above, but with a finer, 3 mm stepsize, yielding a 9 \(\times\) 9 measurement grid. Hall voltages up to 43 \(\mu\)V were observed. Figure 2(a) shows our results, with the theoretical results of Ref. 3 shown in Fig. 2(b).

Figure 1(a) shows the gross features predicted for the calculated Hall weighting function\textsuperscript{8} for the square geometry. It has no regions of negative weighting (unlike the resistive weighting function\textsuperscript{1,11}), a maximum value in the center, and a zero value at the edges of the specimen. Figure 2(a) shows the gross features features predicted for the Hall weighting function for a cloverleaf,\textsuperscript{8} having a larger maximum in the center of the sample decreasing more rapidly to zero as one leaves the center. [As previously, negative values in Fig. 2(b) are the result of numerical roundoff.]

On the other hand, the agreement is only qualitative. The theoretical weighting function for the square [Fig. 1(b)] has its maximum at the center of the specimen, but also along both diagonals. The effect of the finite size of the magnet probe is to wash out the weighting function, and this is most pronounced along the diagonals near the corners. Also,
theory predicts that a cloverleaf having slits equal to a third of its size [Fig. 2(b)] should have a weighting function about six times larger than for the uncut clover, while we see an increase of only a factor of 4. Again, the probe is the probable cause of this discrepancy.

While the agreement between these results and theory is quite rough, it does point out several important features predicted for the Hall weighting function of both the square and the cloverleaf, and proves that resistivity and Hall measurements sample the same specimen differently. Both of these functions are purely nonnegative if contacts are at the edge of the specimen, with a maximum in the center of the specimen. Turning a square into a cloverleaf focuses the measurement on a smaller region of the specimen, as suggested by

van der Pauw. Further work is needed to increase the precision with which one can measure the Hall weighting function, and to test other specimen geometries.