Contact placement errors for resistive and Hall measurements on cross-shaped samples

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The calculation of lead-placement errors for resistivity and Hall signals is extended to the family of cross-shaped samples. Empirical expressions for errors $\delta \rho / \rho = -\frac{1}{2} (2d / D)^{2n}$ and $\delta R_H / R_H = -\frac{1}{2} (2d / D)^{n}$ are given in the limit of large contact displacement $(d / D \to 1)$. For crosses, the exponent $n$ was found to increase as $n \propto (l / D)^{1.42}$ as the ratio of leg width to overall size, $l / D$, was reduced. For square and circular cloverleafs, $n$ is less sensitive to the increase of the slit length, giving power laws with exponents of 0.8 and 0.6, respectively, in terms of the parameter $\lambda = 2r / D$.

I. INTRODUCTION

Systematic errors in experiments on single samples can become random errors when data is taken from more than one sample. Measurement error of charge transport quantities due to size or placement of current or voltage leads is such an error. Contact-placement errors have been studied for van der Pauw (VDP) measurements of resistivity $\rho$ and Hall coefficient $R_H$ for circular disks, for square disks, and for circular and square cloverleaf samples. It has been shown that the choice of square rather than circular sample shape can typically reduce measurement errors by a factor of about 10 (Refs. 2 and 3) (for resistivity errors of about 10%), while the choice of cloverleaf shape can reduce this source of error even more dramatically.

Since the VDP technique is not the only technique for studying $\rho$ and $R_H$, one needs to see how sensitive other techniques, like the bridge technique (Fig. 1), are to contact-placement errors. The bridge technique has the apparent advantage over VDP that, if the material of the sample is inhomogeneous in either thickness or transport characteristics, then the material in a relatively small region in the very center of the sample is weighted disproportionately by the measurement process, thus decreasing the measured sample inhomogeneity. On the other hand, the bridge technique has the disadvantage, relative to the VDP geometry, that the conversion from measured resistances to charge transport quantities depends intimately on the geometry (i.e., the channel width, the distance between probes, and the sample thickness), while the corresponding conversion for a VDP measurement depends only on the sample thickness.

If the bridge technique does indeed confine transport measurement to a small portion of the sample, it would be useful to perform VDP measurements with a bridgelike geometry, to combine that advantage of the bridge technique with the advantages of the VDP technique. This could be achieved with the cross geometry (Fig. 2), which resembles a symmetric, four-legged bridge sample. The differences between the cross and the bridge sample are as follows:

1. There are only four legs on the cross geometry, rather than the usual five or more used for bridge measurements—this prevents one from using a potentiometer to "null out" the zero-field Hall voltage.

2. The location of the current leads in the cross is not fixed.

3. The resistive measurements in the cross are not made with the current flowing the length of the sample, but between adjacent leads.

Further study of the cross should be undertaken to determine how strongly the geometry weights the interior of the sample for an inhomogeneous sample. Only then can one appropriately compare the two geometries. For the moment, we will study contact-placement effects to compare the cross-shaped VDP sample with other sample shapes.

II. METHOD

The electrical potential for a cross-shaped sample was approximated by the potential on a grid of lattice points. The

FIG. 1. A bridge-geometry sample. Current and voltage probes are placed inside the circular pads at the edges of the five legs. Current flows between the pads on the extreme left and right sides. To measure resistivity, one measures the voltage drop between the bottom two pads. To measure the Hall coefficient, one measures the difference between the potential on the uppermost pad and the potential on a variable voltage divider between the lower two pads, which has been adjusted to give zero potential difference at zero magnetic field.
FIG. 2. A cross-shaped van der Pauw sample. The text considers the effect of the displacement of the upper contact by a distance $d$ from the edge of the sample.

potential was numerically iterated to obey Poisson's equation in its interior, and boundary conditions were enforced such that the E field normal to the edge of the sample was zero except at the two points where the current entered and exited the sample.

This iteration was done using a PASCAL language program on an AT&T 6300 computer. The cross shape fit inside a $51 \times 51$ array. The initial potential was zero everywhere except at the current leads. No relaxation schemes were implemented to hurry convergence. The procedure was followed for four different crosses (and a square) representing different widths of the "legs" of the cross. Data were obtained for resistive and Hall geometries for different widths of crosses.

III. RESULTS

The results are shown in Figs. 3 and 4 for errors in resistivity and Hall coefficient, respectively. The $l/D = 1$ sample is a square with leads attached midway between the corners. For this sample, the measurement errors are comparable to the corresponding errors for a circular sample. While errors are smaller for all crosses than for squares with leads attached at the sides, the resistive errors for a square with leads attached at the corners correspond to the errors for a cross with $l/D$ equal to about 0.4. It should be possible to reduce the measurement errors for the cross, for sufficiently small $d/D$, by placing the leads near the corners of the legs of the cross, as shown in Fig. 5.

van der Pauw expressed the small-$d$ limit ($d/D \ll \frac{1}{2}$) of measurement error in the form

![FIG. 3. Resistive measurement error, $\Delta \rho/\rho$, vs the relative displacement, $d/D$, of one contact lead from the edge of crosses of different leg widths, $l/D$. Parameters of the cross are shown in the inset.](image)

![FIG. 4. Hall measurement error, $-\Delta R_H/R_H$, vs the relative displacement, $d/D$, of one contact lead from the edge of crosses of different leg widths, $l/D$. Parameters of the cross are as shown in the inset in Fig. 3.](image)

![FIG. 5. Recommended lead displacement for minimal contact-placement errors.](image)
FIG. 6. A contact lead displaced from the edge of a cross with $l/D = 0.059$ (top) and displaced from the vertex of a 5.8° angle (bottom). In both cases, the resistivity error increases as approximately the 60th power of the displacement from the left-hand side of the figure, and the Hall error as the 30th power.

$$\Delta \rho/\rho = -A(d/D)^{2n}$$

and

$$\Delta R_H/R_H = -B(d/D)^n,$$

where $d/D$ is the fractional displacement of the lead from the edge of the sample and $A$, $B$, and $n$ are some constants. For circular samples,$^1$ $n = 1$ for small $d/D$ and for square samples,$^2$ $n = 2$. For cloverleaf and cross samples, however, the large-$d$ limit $(d/D \rightarrow 1)$ is more pertinent because the small-$d$ limit corresponds to extremely small errors. For crosses, we can represent that limit as in Eq. (1), but with the exponent $n$ equal to a function of leg width, $l$. In this limit,

$$\Delta \rho/\rho = -\frac{1}{2}(2d/D)^{2n}$$

and

$$\Delta R_H/R_H = -\frac{1}{2}(2d/D)^n.$$  

Using the data from Figs. 3 and 4, we find that

$$n = (0.56 \pm 0.06)(l/D)^{-1.42 \pm 0.07}$$

(3)

gives an excellent fit to the data for cross-shaped samples in the large-$d$ limit.

Similar results can be obtained for square and circular cloverleafs in the large-$d$ limit from experimental data.$^3$ The two sets of available experimental data for each geometry give the following empirical equations:

$$n(\lambda) = 1.3\lambda^{-0.8}$$

and

$$n(\lambda) = 1.2\lambda^{-0.6}$$

(4)

for the square and circular cloverleafs, respectively, where $\lambda$ is a parameter that equals 1 for an unclovered square or circle and equals 0 for a cloverleaf in which all four slits meet in the center of the sample. (This parameter is equal to $2r/D$ in the language of Ref. 2—the ratio of the radius of the internal, unclovered region to the total sample radius.) Equations (4) can be substituted into Eq. (2) if $D$ represents the diameter or diagonal length of the sample and $d$ represents the displacement of one contact lead from either the circumference of the circle or the corner of the square. (This definition is different than the definition of $D$ given in Ref. 2, but allows us to define things such that a contact lead in the center of a sample has a displacement $d/D = 1$, regardless of sample shape.)

We can observe that the parameter $l/D$ and the parameter $\lambda$ are both measures of the width of the narrowest constrictions between the current leads and the voltage probes. As such, the exponents in Eqs. (3) and (4) provide a useful way of comparing the sensitivity of contact placement errors to the constriction of each geometry. In this analysis, a sufficiently narrow cross ($l/D \lessgtr 1$) is more effective at reducing contact placement errors than an equally constrictive ($\lambda \lessgtr l/D$) square or circular cloverleaf.

It is interesting to note that if one maps a corner of angle $\theta$ onto the upper half-plane or the unit circle, one uses a local conformal mapping that approximates $u = z^n$, where $n = 180^\circ/\theta$. A lead displaced an infinitesimal distance, $d$, from this corner will map onto a point a distance proportional to $d^n$ away from the edge of the unit circle. For this displaced contact, $\Delta R_H/R_H \propto d^n$ and $\Delta \rho/\rho \propto d^{2n}$. On the other hand, if we know the exponent $n$ in the expression $\Delta R_H \propto d^n$, we can estimate $\theta$, the angle of the “corner” the contact appears to be displaced from. For a cross with $d/D \approx 1/2$,

$$\theta = 180^\circ/n = 320^\circ(l/D)^{1.42}$$

(5)

for sufficiently large contact displacements. This angle equals 5.8° for the cross in this study with $l/D = 0.059$. Conversely, one must admit that as one gets sufficiently close to the edge of one of the cross legs, $n$ approaches 1, as in the circular disk. This is illustrated in Fig. 6.

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