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Rayleigh Scattering: *Why the sky is blue*

Consider the forces acting on the electron cloud of a molecule as the E-field from an electromagnetic comes marching by:

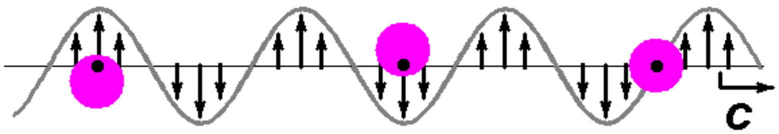

Fig. 1: A "snapshot" of an electromagnetic wave marching past some [circular] molecules. Pink represents electron cloud surrounding black nucleus. The local E-field from the electromagnetic wave shifts the electron cloud in the direction opposite *E*.

$$\sum \vec{F} = m\vec{a}$$

$$-e\vec{E} \quad -k\vec{X} \quad -\gamma\dot{\vec{X}} \quad = m\ddot{\vec{X}}$$

Coulomb Restoring Damping

(What physically causes the Restoring term?)

Since the local $\vec{\mathrm{E}}$ - field varies as $\vec{E} = E_0\hat{i}e^{i(kr-\omega t)} = E_0\hat{i}e^{-i\omega t}$, let's assume that

$\vec{X} = X_0\hat{i}e^{-i\omega t}$. Consequently,

$\dot{\vec{X}} = -i\omega\vec{X}$ and $\ddot{\vec{X}} = -\omega^2\vec{X}$

Let's also set $k = m\omega_0^2$, as with the mechanical oscillator. So then, plugging into the differential equation above,

$$-e\vec{E} - m\omega_0^2\vec{X} + i\omega\gamma\vec{X} = -m\omega^2\vec{X}$$

$$\vec{X} = \frac{-e\vec{E}}{m\omega_0^2 - m\omega^2 - i\omega\gamma} \quad \text{and} \quad \ddot{\vec{X}} = \frac{e\omega^2\vec{E}}{m\omega_0^2 - m\omega^2 - i\omega\gamma}$$

Notice that ω_0 is the molecule's natural [angular] frequency of oscillation, a constant, while ω is the angular frequency of the incoming radiation, which can be varied.

Now, $-e\vec{X}$ represents a charge distribution, which produces a constant dipole E-field.
$-e\dot{\vec{X}}$ represents a current density, which produces a constant B-field.
$-e\ddot{\vec{X}}$ represents a varying current, which produces a time-varying B-field,
which produces a time-varying E-field,
which produces, ..., etc.

So electromagnetic waves are produced which are centered, like ripples, at the molecule, and which scatter the incident wave from its initial direction.
One can look up the formulae for the electric and magnetic fields due to an accelerating charge distribution: $\left\{\vec{B} = \frac{e}{c^2 r}\hat{r}\times\ddot{\vec{X}}; \quad \vec{E} = \frac{e}{c^2 r}\hat{r}\times(\hat{r}\times\ddot{\vec{X}})\right\}$, both of which are proportional to

$$\frac{\omega^2}{m\omega_0^2 - m\omega^2 - i\omega\gamma}.$$

For $\omega << \omega_0$, $E \propto \omega^2$, and so the intensity, S, the rate of energy flow, is proportional to

$$S \propto E^2 \propto \frac{\omega^4}{\left(m\omega_0^2 - m\omega^2\right)^2 + (\omega\gamma)^2} \approx \omega^4 \propto 1/\lambda^4 \text{, and so...}$$

Rayleigh scattering happens for $\omega << \omega$. Near $\omega \approx \omega_0$, (the UV for atmospheric Rayleigh scattering) there would be a ton of scattering if this system behaved like a simple driven, damped harmonic oscillator, as shown to the right. (What really happens is a different type of scattering, namely Mie scattering, but that's another story...)