WEEK 14: Go to class 27, 28

Homework assignments

CLASS 27:

The message from this material is the following:

Electrostatics + Special relativity = Magnetostatics.

We will split this material into two classes: the first class we will look at how an electric field differs when measured by different observers. In the second class we will look at a very specific example, a charge moving parallel to a current carrying wire. We will observe the charge from three different inertial reference frames, considering only electrostatics forces, and conclude that there is a force acting on the charge which is identical to what we identify in the lab as the magnetic force.

GAUSS' LAW FOR DIFFERENT INERTIAL REFERENCE FRAMES

We will begin with two experimentally observed facts: (1) A charge in an electric field experiences a force, $\vec{F} = Q\vec{E}$, and (2) Electric charge is relativistically invariant. (Remember that one from our first day of class?) Next, we will apply an important theorem from electrostatics which should still be true, even for a moving observer, since relativity tells us that the laws of physics are the same for all inertial reference frames. That theorem is the Gauss' Law -- $\int \vec{E} \cdot d\vec{a} = 4\pi k Q_{encl}$

Gauss' law will be true for any electric field, but we will look at a particular example, the parallel plate capacitor, for which it is very easy to apply. If we have a square box that just fits around *one* of the two capacitor plates, then we have $EA = 4\pi kQ$, or $E = 4\pi kQ / A$

There are two cases that interest us here. The parallel plate capacitor may be moving in a direction parallel to the plates, or in a direction perpendicular to the plates. Since the electric field between the plates is the perpendicular to the plates themselves, this means that the electric fields are either perpendicular to or parallel to, respectively, the direction of travel. We will denote these as $E_\perp$ and $E_{||}$, the perpendicular and parallel components of the electric field, respectively.

If the motion of the plates is parallel to the plates themselves ($\vec{E}$ perp. to $\vec{v}$), then there will be length contraction of the plates in the direction of motion. Since *Q* is invariant, and since the area is decreasing by a factor of γ, the electric field will be larger by a factor of γ. That is,

$$E'_\perp = \gamma E_\perp$$

If the motion of the plates is perpendicular to the plane of the plates ($\vec{E}$ parallel to $\vec{v}$), then there is no length contraction of the area of the plates, and so the electric field is the same for both observers:

$$E'_\parallel = E_\parallel$$

An interesting exercise (which we will do in class) is to look at how the other quantities of the capacitor change or don't change under this relativistic transformation. We know that charge remains the same, but what about the voltage between the two plates and the capacitance of the capacitor itself? We'll look at these in class, but this might be a good opportunity for you to work out the answer to this question yourself, to test yourself to see whether not you understand the relativistic transformations we just did.

THE FIELD OF A MOVING CHARGE

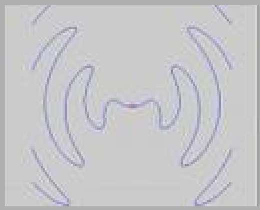
What we've just done for the parallel plate capacitors is in fact true of all electric fields, regardless of what their source is. Consider a point charge. In a frame moving alongside it, the electric field lines stick out at equal angles from the point. In a frame moving relative to the charge (or vice versa), the component of the electric field perpendicular to the direction of motion is enhanced by a factor of γ. The *E*-field lines will appear to flatten out (bunching up toward the vertical), forming an ellipse (its longer axis in the vertical direction), rather than a circular halo around the charge. In fact, any *E*-field line in the point charge's rest frame will transform to another *E*-field line in the moving frame, at a different angle (relative to the direction of motion), given by

$$\tan\theta' = E'_{\perp} / E'_{\parallel} = \gamma E_{\perp} / E_{\parallel} = \gamma \tan\theta$$

ABRUPT AND ACCELERATED CHANGES IN VELOCITY

Electric field lines propagate at the speed of light. This means that if a charge was moving relativistically along a line and then abruptly stopped (maybe it collided into another charge) there's a portion of the universe that doesn't know about it yet. If this happened a time *Δt* ago, then, outside a sphere of radius *cΔt*, the electric field lines still look like the ellipse-shaped lines we described above. Inside that sphere, however, the electric field lines will appear as they do for a charge at rest: they will be radiating out at equal angles from the charge's location. On the other hand, the electric field lines outside the sphere will be centered on the location where the charge would be *if it hadn't stopped*. We will look at a wide variety of just this kind of example. You will be asked to sketch the electric field lines of a charge that either moves and stops, or begins at rest and suddenly moves. You will also be asked to decipher electric field lines, giving a quantitative description of what kind of charge was doing what, when.

By the way, a very interesting case arises when we have a charge oscillating back and forth. If its oscillation is along the *x* axis, the electric field along the *y* axis will look like a sine wave, while the field line in the *x* axis will be a straight line. This corresponds to a radiating dipole, producing the sort of electromagnetic radiation produced by linear radio antennas (which we just studied last week).



Accelerating charge - radiation
http://www.youtube.com/watch?v=YGCyvlppII4

CLASS 28:

THE FORCE ON A MOVING CHARGE NEAR A CURRENT-CARRYING WIRE

This is a very messy, algebra-intensive problem in the text. Study it carefully. We will study it at length in class, because I think is very important to see it. The results are very elegant: they show that by adding special relativity to electrostatics, we get magnetostatics. In other words, magnetism is a relativistic phenomenon. This blew me away when I first saw it (and I was teaching E&M at the time). We are used to thinking of relativity and as being totally irrelevant to our lives, because, for objects traveling at speeds we are familiar with, length contraction and time dilation are tiny. It is only objects traveling close to the speed of light that contract or dilate. But here we have microscopic charges travelling at "non relativistic" speeds producing sizable effects that *we can measure*. The magnets on your refrigerator are proof of Einstein's theory of relativity!

I won't try to recreate the mathematical derivation in these notes. But I will try to give you the "big picture" of it. First, we draw a wire, containing free "conduction" electrons and the positive charges left behind by these electrons. Consider the "lab frame": the frame in which the wire is not moving. The average distance between conduction electrons is the same as the average distance between the ions, and so each type of charge has the same charge per length along the wire. Now we consider a "test" charge some distance away from the wire, traveling with a velocity parallel to the wire. In the lab frame, this test charge doesn't appear to experience a net force from the charges in the wire. We conclude that the net electrostatic force on the test charge is zero. We do notice, however, that the conduction electrons, since they are moving, are not in their own rest frame. The distance between the conduction electrons is relativistically contracted, relative to their own rest frame, and so the charge density in their rest frame is smaller than in the lab frame.

Now, consider this picture from the *test charge's* rest frame. In order to calculate the net electrostatic force on this charge due to the conduction electrons and ions in the wire, we need to calculate the relativistic contraction of the average distance between conduction electrons, and the average relativistically contracted distance between ions. This is where things get ugly, especially because in order to calculate the conduciton electrons' velocity in this frame, we have to use the relativistic velocity addition formula. At the end of a long pile of ugly algebra, we see that in this frame, the test charge experiences a net nonzero electric force.

The final step is to transform this force back into the lab frame, since that is the frame in which we are measuring things. When we do this, we find it that the point charge experiences a force which is exactly equal to what we empirically (experimentally) call the magnetic force

$$\vec{F} = Q\vec{v} \times \vec{B}$$

Assuming that the current-carrying wire produces a magnetic field, $\vec{B}$, of magnitude $\mu_0 I / 2\pi r$. Wow. Again, the main point of this exercise is not the algebra, nor to practice solving problems of this type, but to demonstrate that

a) Magnetism is a relativistic effect.
b) It is impossible to have electrostatics without magnetism in a universe in which Einsteinian relativity is true. (or to have electrostatics and magnetism without relativity being true)
c) What appears in the moving charge's rest frame to be an electric force is in the lab frame a magnetic force: electric and magnetic fields transform into each other when one switches between inertial reference frames. There is a set of transformation equations for $\vec{E}$ and $\vec{B}$ not unlike the Lorentz transformations between position and time.

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