WEEK 12: Go to class 23, 24

Homework assignments

CLASS 23:

SECTION 7.3: MAXWELL'S EQUATIONS
FIXING UP AN INCONSISTENCY:
There is an inconsistency in the laws of electric and magnetic fields as they now stand. If you take $div(curl\vec{E})$, you get the time derivative of $div(-\vec{B})$, and both sides should be equal to zero, which they are. Notice, as the book points out, and as you are asked to as a daily assignment, that the divergence of the curl of any normally-behaved mathematical vector function is zero. $div(curl\vec{A}) = 0$.

But now notice that if you take $div\left(\vec{\nabla}\times\vec{B} = \mu_0\vec{J}\right)$, the left-hand side ought to be zero and the right hand side will be $\mu_0 div\left(\vec{J}\right)$. But $div\left(\vec{J}\right)$ is only zero if the charge density, *ρ*, is constant. Maxwell set out to fix this. And he fixed it by a just adding another term to $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$:

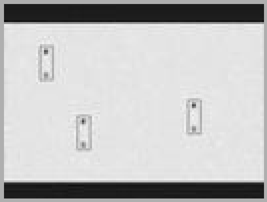
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\dot{\vec{E}}$$

OK, Maxwell's kludge (i.e. "hack") fixes the mathematics, but what does it *mean*? Notice how this expression looks a lot like Faraday's law, $\vec{\nabla}\times\vec{E} = -\dot{\vec{B}}$. The righthand term in that expression means that a change in magnetic field can produce an electric field which curls around of the magnetic field lines. By analogy, we would expect that if we have an electric field which is varying with time, this will produce a magnetic field lines that curl around the electric field lines. What is really cool about this is that if we have electric fields which are varying sinusoidally with time, they will produce magnetic fields also varying sinusoidally with time, etc. That is, a varying field can produce other varying fields ad infinitum. This will give rise to electromagnetic radiation, which we will look at more closely in Chapter 9. In fact, this was the big breakthrough of Maxwell's work: he predicted the existence of electromagnetic waves. Now this is kind of like J. J. Thomson's discovery of the electron. Edison had already been harnessing electrons for decades in various electric inventions without understanding the nature of the electricity which he was exploiting. Same thing with electromagnetic waves. Optics is all about electromagnetic waves. Visible light, infrared, ultraviolet, radio, and other such waves are electromagnetic waves.

The experimental breakthrough came with Heinrich Hertz, who discovered that he could produce a spark on one end of the room which could be detected on the opposite side of the room. This led to the exploitation of radio waves by Guglielmo Marconi, which made it possible to communicate across the Atlantic without waiting for a ship to make the trip. The really amazing thing about all this is that Maxwell predicted the velocity of these waves, and it is a function of the permittivity and permeability of free space, and it is exactly equal to the speed of light which had already been measured centuries earlier by Ole Rømer.

MORE SYMMETRICAL VERSIONS OF MAXWELL'S EQUATIONS:
We can now sum up what we know about electromagnetics with four equations, **"*Maxwell's* equations**" (even though Maxwell could only whittle it down to 20 equations in 20 unknowns).

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \qquad \vec{\nabla}\times\vec{E} = -\dot{\vec{B}}$$
$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\dot{\vec{E}}$$



Del Cross B (song)
http://www.youtube.com/watch?v=igcr5uPt5Hg

These equations are *very nearly* symmetric. The symmetry is destroyed by the apparent lack of magnetic charges ("magnetic *monopoles*") in the Universe (our present understanding (as of 1998), no-monopole song, more Maxwell Equation music). We can make the symmetry more obvious in either of two ways:

(a) First consider **Maxwell's equations in free space** (where there are no charges or currents available)

$$\vec{\nabla}\cdot\vec{E}=0 \quad \vec{\nabla}\times\vec{E}=-\dot{\vec{B}}$$
$$\vec{\nabla}\cdot\vec{B}=0 \quad \vec{\nabla}\times\vec{B}=+\mu_0\varepsilon_0\dot{\vec{E}}$$

(b) Next consider what **Maxwell's equations** might look like **if magnetic charges existed**:

$$\vec{\nabla}\cdot\vec{E}=\rho/\varepsilon_0 \quad \vec{\nabla}\times\vec{E}=-\mu_0\vec{J}_m-\dot{\vec{B}}$$
$$\vec{\nabla}\cdot\vec{B}=\mu_0\rho_m \quad \vec{\nabla}\times\vec{B}=\mu_0\vec{J}_e+\mu_0\varepsilon_0\dot{\vec{E}}$$

where ρ_m is the density of *magnetic* charge, and $\vec{J}_m$ is the *magnetic* current, the rate at which *magnetic* charge flows. (The units not being symmetrical in the equations above is just a sad consequence of our choice of units: in CGS units we would set $\varepsilon_0=\mu_0=1$.)

MAXWELL'S EQUATIONS INSIDE MATTER:

Finally, let's consider what happens inside materials. The big difference is that there are bound *charges* (for *electric* fields in matter) and bound *currents* (for *magnetic* fields in matter). This makes it useful to define two new fields, (They were already defined in Chapters 4 and 6, but we didn't have much use for them until now.) $\vec{D}$ and $\vec{H}$. If this seems like a real nuisance, bear this in mind: living with these new fields means that we can replace ρ and $\vec{J}$ with ρ_f and $\vec{J}_f$, the *free* charge and current densities. That is, we won't have to worry about the densities of charge or current *inside* the material of interest (including the bumpy, uneven density of charge within the atom itself), densities that are responses to the fields we impose from outside, and densities that *we cannot directly change*.

$$\vec{\nabla}\cdot\vec{D}=\rho_f \quad \vec{\nabla}\times\vec{E}=-\dot{\vec{B}}$$
$$\vec{\nabla}\cdot\vec{B}=0 \quad \vec{\nabla}\times\vec{H}=\vec{J}_f+\dot{\vec{D}}$$

All that we are actually going to do with this final version of Maxwell's Equations is to use it to do some interesting optics. It is this variation of Maxwell's equations that causes much of optics that we have taken for granted up till now: the laws of reflection and of refraction in transparent media. We will see *why* light bends as it passes between material, and why it reflects at an angle equal to the incident angle, and find out that it is because of electrodynamics. But that will have to wait for Chapter 9.

FINAL REVIEW SINGALONG

CLASS 24:

SECTION 9.2.1: DECOUPLING MAXWELL'S EQUATIONS

$$u = e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

Musical derivation of the wave equations
http://www.youtube.com/watch?v=YLlvGh6aEIs

All right, we have Maxwell's equations, now what do we do with them? Let's start by calculating $curl\left(curl\left(\vec{E}\right)\right)$. After a couple of steps -- including the one that stars in the "Daily" for next time -- we arrive at an expression that shows that each component of the electric field obeys the wave equation. Repeating this procedure for $curl\left(curl\left(\vec{B}\right)\right)$, we get a similar result for the magnetic field. What we also get is the result that the velocity of the electric and magnetic waves is the same, $v = \left(\mu_0 \varepsilon_0\right)^{-1} = 2.998 \times 10^8 m/s$, a number that should look quite familiar: it is the speed of light in a vacuum. Coincidence?

We will go over the mathematics of this in class, which begin with Maxwell's equations in free space:

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \vec{\nabla}\times\vec{E} = -\dot{\vec{B}}$$
$$\vec{\nabla}\cdot\vec{B} = 0 \quad \vec{\nabla}\times\vec{B} = +\mu_0\varepsilon_0\dot{\vec{E}}$$

We will also show that the equations we obtain are consistent with the wave equation, and we will look at a few functional forms that are valid solutions to that equation.

SECTION 9.2.1: *RE*COUPLING MAXWELL'S EQUATIONS

We have what appear to be six independent wave equations, the x-, y-, and z- components of $\vec{E}$ and $\vec{B}$. However, these six are not totally independent of each other. We simplify things (really!) by defining the *z*-axis to be the direction of propagation of the electromagnetic wave. Then, assuming reasonably arbitrary forms for *E* and *B*, we apply the two curl equations from Maxwell's equations, we see how the electric and magnetic fields are related.

The chief result of this exercise, which I will ask you guys to do on the board in groups of two, will be (a) to show that the electric and magnetic fields are transverse to the motion (and result in two independent polarizations), (b) to find the relation between *ω*, the angular frequency of the wave, and k, the wavevector, and (c) to derive the relationship between the magnitudes of the electric and magnetic fields, namely that $B_0 = E_0 / c$. (In CGS units, $B_0 = E_0$.)

9.2.3: ENERGY AND MOMENTUM

While the expressions for the electric field components of a wave give us all the information we need, they're not the most useful quantities for describing these waves. More useful is the rate at which energy is travelling through space on the crest of that wave. We already have expressions for the energy density in an electric or magnetic field. The first thing we will show is that, given the relationship between B_0 and E_0, electromagnetic waves split their energy evenly between their electric and magnetic fields components.

Next we can define a vector which gives the intensity of the electromagnetic wave and tells us the direction in which it is poynting. This is known as the Poynting vector (after John Henry Poynting, 1852-1914). It has units of power per area, exactly the sort of units that you would need in order to determine whether not there's enough light in the room to read your text by. The expression for the Poynting vector is

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Notice that, for electromagnetic waves in free space, $S = cu$, where *u* is the energy density of the electromagnetic field.

I want to introduce one last expression for a property of the electromagnetic wave: radiation pressure. This is nothing more than the force per area, $P = F / A$, exerted by the wave. You may be familiar with science fiction or speculative fiction in which people talk about building enormous solar sails to transport spacecraft. The question is how large a force does light incident on such a sail exert on it. From relativity we know that $p = E / c$ for photons because they travel at the speed of light. Consequently, we can show that the radiation pressure is $P = u$, that is, the radiation pressure exactly equals the energy density. Compare this to the Bernoulli principle in classical fluid dynamics.

By the way, this radiation pressure does *not* explain how a radiometer works, but *does* explain how a solar sail does.

THE 3K BACKGROUND RADIATION

At this point, we'll take a brief break from our book to talk about the background microwave radiation from the Big Bang, placing it in its scientific and historical perspective. This microwave background radiation is the main proof we have of the Big Bang and the age of the Universe. Check out this site, plus I'll provide a handout that tells a little bit of the history of the 'discovery' of this phenomenon, or rather how the scientists came to understand what it was they were seeing.

Go to YSBATs