WEEK 9: Go to class 17, 18

Homework assignments

CLASS 17:

AMPÈRE'S LAW

the law of Biot and Savart is even worse than it appears. Notice that the curly *r* in the denominator, as well as the curly *r* unit vector in the numerator both vary as you integrate. If only there were a shortcut like Gauss' law gives us. Fortunately there is --- Ampère's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

This integral is a closed path integral. That means that you integrate over a path which is a closed path --- a path that returns to its origin. Now $I_{enclosed}$ is the total current which passes through a surface whose edges are defined by the closed path. For example, consider a wire of circular cross section. If the path of integration were a circle going around the circumference of the wire, then I_{enclosed} would just be the current in that wire. As a practice, confirm to yourself that Ampère's law is true for the magnetic field produced by a long straight wire at a distance r from the axis, where r is greater than the radius of the wire itself. The trick in applying Ampere's law, just like the trick for applying Gauss' law, is to choose an appropriate integration surface. For this wire, the obvious choice is a circle centered on the axis of the wire itself.

DIFFERENTIAL FORM OF AMPÈRE'S LAW

In the same way that we can use Gauss' theorem to come up with a differential form of Gauss' law, we can use Stokes' theorem to come up with a differential form of Ampère's Law:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

where the righthand side is defined below…

CURRENT DENSITY, $\vec{J}$

Remember how we had three fundamental quantities in electrostatics --- *ρ*, $\vec{E}$, *V*? We can define three analogous quantities for magnetostatics. Obviously, $\vec{B}$ is the analogue for $\vec{E}$. In this class, we will ignore the magnetic potential, partly because it is, alas, a *vector* potential. But what is the analogue for the charge density? Well, since all electrostatics fields are produced by charges, and all magnetostatic fields are produced by currents, there ought to be some kind of "current density". The difference between it and the charge density how, however, is that the current density is a density of current per area, rather than per unit volume. The reason for this is that there is no such thing as a "point current". (Think about that.) We use the vector $\vec{J}$ to describe this current density. But current and charge are also related to each other. This relation can be summed up, in differential form, by what is known as the Continuity Equation:

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

where *ρ* is the charge density. Convince yourself that this makes qualitative sense. (What would the case in which the righthand side does not equal zero represent?) In steady state, the righthand side vanishes: *div* $\vec{J}$ = 0.

DIVERGENCE OF $\vec{B}$

We saw above that Ampère's Law in differential form gives us an expression for the curl of $\vec{B}$. It is observed experimentally that the divergence of the magnetostatic field is zero.

$$\vec{\nabla} \cdot \vec{B} = 0$$

Now, remembering that the divergence of the electrostatic field was proportional to the charge density, we can interpret this expression as telling us that there is no magnetic equivalence of a point charge -- which would correspond to a "point current". Such an equivalent, which we would call a "magnetic monopole", simply does not appear to exist in our universe. As it happens, there are a number of theorists who have come up with theories which demand the existence of magnetic monopoles. That's the good news as far as the existence of magnetic monopoles goes. The bad news is that the theories I've heard of posit the existence of exactly one such magnetic monopoles in our Universe. How does this affect you personally? Well, the odds of you ever running into that one hypothetical magnetic monopole in the course of your brief existence in this out of the way corner of the universe is about as minimal as that of your winning an all-expense weekend trip to Alpha Centauri. So, even if *div* $\vec{B}$ is not strictly equal to zero, for your purposes it is convenient to pretend that it is.

COMPARE AND CONTRAST: ELECTROSTATICS v. MAGNETOSTATICS

All right, let's began with a comparison between electrostatics and magnetostatics. First of all, electric *charges* produce electric fields. Electric *currents* produce magnetic fields.

COULOMB, BIOT AND SAVART: $\vec{E} = \frac{kq\hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2} = \frac{1}{4\pi\varepsilon_0}\frac{q\hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2}$ $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2}$

Second, Coulomb's law describes how a *point charge* produces an electric field. The law of Biot and Savart describes how a *point current* produces a magnetic field. The problem with using the law of Biot and Savart is twofold: first of all, it involves a cross product, that is, unlike Coulomb's law, the magnetic field is not parallel to the position vector. Second of all, there is *no such thing* as a point current. It is necessary to integrate over the entire current loop in order to calculate the total magnetic field.

LORENTZ FORCE: $\vec{F} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Next, the force produced by a magnetic field is not parallel to the field itself, as is the case for a electric forces and fields. Again, there is a cross product, which complicates the mathematics.

GAUSS and AMPERE: $\oint \vec{E} \cdot d\vec{a} = 4\pi k Q_{enclosed}$, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$

Now, since Coulomb's law and the law of Biot and Savart are both "formal definitions" (i.e. "practically worthless"), we need to use something else to solve for the field. For electrostatics, we use *Gauss' Law*, for magnetostatics, we use *Ampere's Law*. In Gauss' Law, the left-hand side is a closed *surface* integral, with a *dot product* between the field and the increment. In Ampere's Law, the left-hand side is a closed *line* integral, with a *cross product* between the field and the increment. In Gauss' Law, the right hand side is proportional to the enclosed *charge*. In Ampere's Law, the right hand side is proportional to the enclosed *current*. Notice, once again, that the electric field is created by charge and the magnetic field is created by current.

DIFFERENTIAL FORMS OF GAUSS AND AMPERE: $\vec{\nabla}\cdot\vec{E} = 4\pi k\rho = \frac{\rho}{\varepsilon_0}$ $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Let's compare the differential forms of Gauss and Ampere. The differential form of Gauss' Law relates the *divergence* of the electric field to the *charge* density. The differential form of Ampere's Law relates the *curl* of the magnetic field to the local *current* density. Again, ***electric = charge, magnetic = current***.

CURL AND DIVERGENCE $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\cdot\vec{B} = 0$

We looked at two of the vector differentials of the two types of fields. Let's look at the other two vector differentials. We saw that the *curl* of the electric field is zero. This means that the electric force is conservative. Now we see that the *divergence* of the magnetic field is zero. This means that there are no magnetic monopoles.

These four vector equations for the electric and magnetic field give us what we will call Maxwell's equations, but we will need to fix them up a little in a later chapter.

LAW OF CONTINUITY: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} = -\dot{\rho}$

OK, now for some new stuff. First of all, electric and magnetic fields are related because their sources, the electric charge and electric current, are related. And this is shown in the Law of Continuity. If there is a point source of current, it comes at the expense of sucking current out of that point.

CLASS 18

SECTION 7.1: ELECTROMOTIVE FORCE

OHM'S LAW: $V = IR$, $\vec{J} = \sigma\vec{E}$, $\vec{E} = \rho\vec{J}$

When current flows in an object, the voltage drop along the length of the object is usually proportional to the current. This is an experimental fact for many materials in specific ranges. It *doesn't* work for diodes, or for objects like tungsten filaments for which the voltage vs current graph changes with the temperature of the object.

This macroscopic equation is like Gauss' Law and Ampere's Law: it can be replaced by an equation that holds (or doesn't) at a point in space. The pointlike analog of current, *I*, is current density, $\vec{J}$, and the pointlike analog of voltage is electric field. The pointlike analog of resistance, *R*, is something we will call resistivity, *ρ*. So the pointlike versions of Ohm's Law are the last two of the three equations given above as "Ohm's Law". Notice that resistivity and conductivity, *σ*, are inverses of each other for simple systems: $\rho = 1/\sigma$.

These microscopic versions of Ohm's Law are not strictly true, even for 'Ohmic' materials. They are true if two conditions hold, namely, first there is no magnetic field, and secondly the material is isotropic. Multicrystalline materials, such as the metal you would find in a wire, *are* isotropic. But for single crystals, we have to be careful. In a more complex model, the conductivity and the resistivity are not simple scalars, but matrices. In fact, the Hall effect, which produces an electric field perpendicular to the flow of current, (We'll discuss it later.) can be thought of as an off-diagonal element in the resistivity matrix.

The book makes another point which I mentioned several chapters ago. Part of my motivation for showing you novel ways of solving Laplace's Equation was my assertion that you can use it to solve electric current flow problems. The book shows that if you start with the continuity equation, $\vec{\nabla} \cdot \vec{J} = -\dot{\rho}$ and consider only steady currents, and assume constant conductivity, you get back $\vec{\nabla} \cdot \vec{E} = 0$, and Laplace's Equation, $\nabla^2 V = 0$. I just mention this to justify what we did months ago.

DRIFT VELOCITY: v_d

What is the microscopic effect of putting a voltage across a conducting material? The voltage sets up an electric field in the material. This *E*-field produces a force on the charge carriers. You would thus expect them to be accelerated down the wire. (You would be wrong.) This would be the case if it weren't for microscopic scatterers in material. We can analyze what happens in a conducting material using a semiclassical model known as the Drude model. A single charge carrier accelerates down the wire -- as a result of the applied voltage -- but then scatters in a random direction after hitting some scatterer in the material. There are two interesting things about this model. First of all, it suggests that if we could take out all those scatterers, we would not have to provide a voltage across our wire to get current to flow. It turns out that there are materials for which this is the case. They are called superconductors. Another interesting thing is that we can calculate the velocity associated with the flow of current. This is called the drift velocity, v_d, and it is superimposed on the thermal motion of the charge carriers.

The average thermal velocity of the charge carriers is zero, but that doesn't mean that they're not moving. In fact, because the atoms have an average kinetic energy associated with its temperature, we would expect that the average magnitude of that thermal velocity would be nonzero. In fact, the drift velocity is much smaller than the average magnitude of the thermal velocity. It's only because the various thermal velocities of the charge carriers average to zero (because they are vectors pointing in random directions) that the small drift velocities of the various charge carriers (which *don't* point randomly, and therefore don't average to zero) produce a net flow of charge, called current, in the wire. Oh, one last thing. The Drude model is called a *semi*classical model only because electrons were not discovered until 1897. We can thus think of electrons as quantized charge, which is what makes them unclassical.

JOULE HEATING: One of the reasons that the microscopic picture of conduction is useful is that it explains why it requires work to move charge in any material other than a superconductor. Because thermal agitation randomizes the direction of the charge carrriers from the direction that the applied voltage is pulling them, this randomizing behaves like a friction. The rate at which energy is lost through this agitation (and any agitation caused by the randomness of scatterers) is given by the expression for Joule heating that you've already learned in Phys 152:

$$P = VI = V^2 / R = I^2 R$$

This is the *macroscopic* form for the heating in a charge-carrying material. Notice that if you are given any two of the four following quantities -- *I, V, R, P* -- you can solve for the other two using Ohm's law and this expression.

One way to observe Joule heating (other than looking at an incandescent light bulb) is to throw a conductor in a microwave oven. Below are two videos. In one, the oscillating electric field produced by the microwave oven causes current to flow along the thin strip of [electrolyte-containing] water holding the two halves of a grape together. In the other, we see what happens when we induce currents in the aluminum disc inside a CD.

Grape microwave plasma
(Note: grape halves act as dipole antenna.)
http://www.youtube.com/watch?v=EX6Tcfi7REM

[yet another video of a]
CD in a microwave oven
http://www.youtube.com/watch?v=EyRnEEKPXg8

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