WEEK 8: Go to class 15, 16

Homework assignments

CLASS 15:

CHAPTER 5: MAGNETOSTATICS

I have yet to find a textbook that treats magnetostatics well. There's a lot of difficult, confusing material which is usually poorly organized. I will try to help out here by providing an alternate way of organizing this material in these lecture notes. Here is what I will do in these lecture notes for this chapter: first I will present the material on magnetostatics which appears in the book, organized in terms of "forces" and "sources" (nice catchy aid to remembering the two categories). In other words, for most practical problems, what you really need to know is how to calculate (a) the magnetic fields caused by a certain arrangements of currents and (b) the magnetic force on electric objects. Second, I will look at the properties of the magnetic field and current density field, using some vector calculus. Thirdly, I will rehash all of this material in order to point out the similarities and differences between the electrostatic and magnetostatic fields. Finally, I will describe two interesting experiments in magnetism that are not in this text.

Conceptually, what do I want you to get out of this material? First, I want you to have a practical handle on calculating magnetic fields and the effects that they have on charges and currents. Secondly, I want you to understand the similarities and differences between electrostatics and magnetostatics fields. More on all of this in the YSBATs.

BACKGROUND

Experimentally, we find that a moving charge located in a region of space containing electric and/or magnetic fields experiences a net force called the Lorentz force:

$$\vec{F} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Notice the first striking difference between the electrostatics and magnetostatics: magnetic force is not parallel to the magnetic field, but perpendicular to it. In order to talk about what causes these magnetic fields, we have to do a little bit of housekeeping here, introducing a a quantity that we should have introduced earlier, an electro*dynamic* quantity which you probably already recognize as the current, *I*, the rate at which charge passes past some point in space:

$$I = Q / t$$

The units of current are Amperes or amps, *A=C/s*. By the way, the SI units of magnetic field are called Teslas: *T=N/(A · m)*. It is a useful exercise to confirm to yourself that this is the combination of units that would be consistent with the expression above for the Lorentz force. We will probably do similar calculations like this in class, just to practice the algebra of checking for units.

MAGNETIC *FORCES*

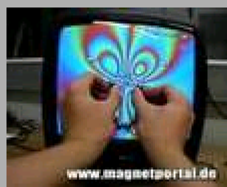
We will introduce two important expressions for (a) on the one hand, the force on a moving charge, and (b) on the other hand, the force on a current-carrying wire. In both cases, you need to memorize two things: (a) the vector equation for the force and (b) the "Right Hand Rule" for determining the direction of the force. When we are done with these two expressions for forces, we will look at a couple of the expressions for magnetic *sources*. Two of these expressions will also have associated Right Hand Rules.

The first expression is for the force on a moving charge. You have actually already seen this, because it is the second term of in the equation above for the Lorentz force. The *magnetic* force on a moving *electric* charge is

$$\vec{F}_B = Q\vec{v} \times \vec{B}$$

Now consider a wire which is carrying electric current. We can picture this as containing a stream of electric charges travelling down the length of the wire with some velocity, *v*. There ought to be some force acting on these charges. One can show that the equation for the force on a moving charge yields this expression for the force on a current-carrying wire:

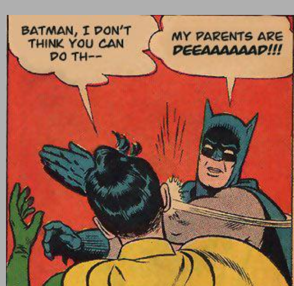
$$F = IlB\sin\theta$$



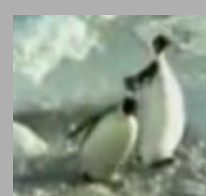
Magnetic deflection of TV tube
http://www.youtube.com/watch?v=s1xS-ssfTM8

Lorentz force on a wire
http://www.youtube.com/watch?v=_X8jKqZVwoI

Now for the "Right Hand Rules". Here, the order in which we write the terms on the right hand side is very important. Since these are cross products, we simply use the right hand rules for a cross product. There are a number of ways of stating the Right Hand Rule, differing from textbook to textbook and instructor to instructor. I will give you my version, but you should feel free to come up with your own expression, as long as it is consistent with mine. If we have a vector $\vec{C} = \vec{A} \times \vec{B}$, you can find the direction of $\vec{C}$ by a pointing your fingers in the direction of $\vec{A}$ and then slapping them in the direction of $\vec{B}$. Now, if you put your fingers pointing in the direction of $\vec{A}$, your palm will be facing in any one of a variety of directions. But there is only way you can point your palm so as to be able to slap in the direction of $\vec{B}$ without straining your wrist (i.e. without slapping more than 90^o). So, after you point our fingers toward $\vec{A}$, you have to orient your palm in such a way that you can slap in the direction of $\vec{B}$. Once you have done all of *that*, you find that your thumb is pointing in the direction of the vector $\vec{C}$.



Incorrect application of righthand rule
http://files-cdn.formspring.me/photos/20120106/n4f06db36057c8.jpg.
Accessed 1/9/12.



Correct application of the righthand rule:
$\vec{A}$ is to our left (penguin's right).
$\vec{B}$ is forward for the penguin.
Therefore $\vec{C}$ points straight up (if penguin had a thumb).
http://www.youtube.com/watch?v=2DB7Jm0m-jU

To recap, to find the direction of the force on a moving charge, point your fingers in the directions of *QV*, and slap in the direction of the magnetic field, $\vec{B}$. To find the direction of force on a current-carrying wire, point your fingers in the direction of the current and then slap in the direction of $\vec{B}$. In either case your right thumb points in the direction of the force. Any of you who are left-handed will have an easier time because you can continue to take notes while your right hand is figuring out the direction of force.

CLASS 16:

MAGNETIC *SOURCES*

The two principal sources of magnetic fields we will discuss are both current carrying wires. One is simply a straight length of wire; the other is a circular loop. At a distance *r* from the axis of the straight wire -- as long as *r* is much less than the length of the wire -- the magnetic field equals

$$B = \frac{\mu_0 I}{2\pi r}$$

where μ_0 is a constant known as the "permeability of free space" which is equal to $\mu_0 = 4\pi \times 10^{-7}\, N/A^2$. It is a useful exercise to confirm that the units on the lefthand and righthand sides of the equation equal.

Notice that this is not a vector equation. This means that we need some way of determining the direction of the field. We will introduce the appropriate Right Hand Rule soon.

Straight long wires are not very useful if you're trying to design your own electromagnet. The magnetic field extends into all of space, and falls off very slowly. But most importantly, you would need a very strong current in order to get any sort of decent magnetic field anywhere except right next to the wire. A much more effective source for the magnetic field is a circular coil. The equation for the field near a circular coil is generally messier than for the field near a long straight wire. However, I will give you an expression for the magnetic field *at the center of the coil*, and then, for your own future reference, an expression for the magnetic field along the axis of the coil. For a coil of radius *R*, the magnetic field in the center equals

$$B = \frac{\mu_0 I}{2R}$$

For that same coil, the magnetic field along its axis, at a distance *z* from the plane of the coil, is given by

$$B = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + z^2\right)^{3/2}}$$

Now a single coil is not a very good electromagnet either. The field drops off quickly along its axis. There are two common ways of getting around this. One is called a solenoid. In this, we wrap wires around a cylindrical form, so that we are simply adding a large number of single coils together, each of them displaced along the *z*-axis from each other. If the solenoid is sufficiently long, then the magnetic field inside the solenoid can be made nearly constant, equal to

$$B = \mu_0 n I = \mu_0 N I / l$$

where *n* = the number of coils per length of the solenoid, *n=N/L*, where *N* = the total number of coils in the solenoid, and *L* = the total length of the solenoid. In order to approach this limit, you need to fulfill the following conditions: the solenoid must be much longer than it is wide, and you must be measuring the magnetic field close to the axis of the solenoid.

Another improvement over the single-coil electromagent is to put two coils (each of radius *R*) a distance R from each other along the same z axis. This arrangement is called a Helmholtz coil. This is particularly useful if you don't need a very large magnetic field, but you need the field to be relatively uniform in the volume surrounding the center of the coils. We'll explore this uniformity in a "Daily" problem.

RIGHT HAND RULES FOR MAGNETIC *SOURCES*
The Right Hand Rules for these two sources --- the long straight wire and the single coil --- are similar to each other but quite different from the right hand rules we developed for magnetic *forces*. Those right hand rules involved slapping; these right hand rules involve holding your right hand with the fingers curled and the thumb pointing straight out. (more like hitchhiking than slapping) The only two quantities involved are the current and the magnetic field. So these Right Hand Rules are fairly easy to remember. If the current is pointing in a straight line, point your thumb in that direction and the fingers will curl in the direction that the magnetic field curls around that wire. If the current is pointing in a loop, curl your fingers in the direction in which the current curls, and your thumb will point in the direction of the magnetic field *inside that loop*.

The hardest part is wrapping your brain around the idea that the B-field surrounding a long, current-carrying wire points in a circle. So let's consider a simple example. Imagine a power line, many feet above the ground, in which the current points east. On the ground, the B-field points north. Above the wire, the B-field point south. At the height of the wire, but south of the wire, the B-field points toward the ground. And at the height of the wire, but north of the wire, the B-field points straight up. In each case, in order to figure out the direction of the B-field, you begin by pointing your thumb in the direction that the current points, namely East, and then putting your fingers in the direction at which you want to measure the direction of the field. If you want to know the direction of B on the ground, for example, point your thumb towards the east with your palm facing north, so that your fingers are below your thumb, and you will see that they are curling towards the north. Work through this example to see whether or not you get it. If not, come to class with a bunchload of questions.

THE LAW OF BIOT AND SAVART
It will seem that we have skirted the real, basic issue here: what is the magnetic analogue of Coulomb's law? The reason we have skirted this is that the equation is much uglier than Coulomb's law. The reason for this is that the source of any magnetostatic fields is a current, and there is no such thing as a "point current", as there is a "point charge". This suggests that in order to calculate a magnetic field, we will always need to integrate something. The other problem is that the magnetic fields created by a current is always perpendicular to the direction of that current. As a result the equivalent of Coulomb's Law for magnetostatics, the Law of Biot and Savart, has the following, ugly form:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

Here I have given you the differential form, unlike the integral which the book gives you, because frankly this is a bit less ugly and it more closely resembles Coulomb’s Law.

Go to YSBATs