***Condensed Story of Ms Farad* by A. P. French**
Miss Farad was pretty and sensual
And charged to a reckless potential;
But a rascal named Ohm
Conducted her home -
Her decline was, alas, exponential.

WEEK 4: Go to class 7, 8
Homework assignments

CLASS 7: EXAM I

CLASS 8:

SECTION 2.5: CONDUCTORS

In our earlier discussion of electric fields, we assumed that we could put charge somewhere and it would stick. But there is a class of materials for which charges don't stay put: these are called conductors. We will start with the easiest conductors to deal with mathematically, namely, perfect conductors, and later (Chapter 7) we will look at what happens in real conductors. The main consequence of applying electric fields near conductors it is that the conductors have free charges which will rearrange themselves until they reach some kind of steady state arrangement. By definition, since these are *free* charges, their steady state configuration would have to be such that the electric field acting on them can no longer move them. Inside the conductor this means that the electric field must be zero, but on the surface of the conductor it possible for there to be an electric field acting on the charge, so long as it points the charge in a direction normal to the surface. As a result of all this, we can deduce the following properties of conductors:

1. $E = 0$ inside.
2. E is perpendicular to the surface just outside the conductor.
3. The charge density $\rho = 0$ inside the conductor.
4. All excess charge in a conductor resides on the surface.
5. The conductor is an "equipotential": every part of it is at the same electric potential, V.

INDUCED CHARGE, Section 2.5.2

Since it is possible for excess charge to reside on the surface, and since the electric field inside the surface is zero, the electric field just outside the surface must be related to the charge density on the surface. To see this, go back to the example we had when we applied Gauss' Law to a 2-dimensional distribution of charge. Applying this example to the general case of the surface charge on a conductor, we get an expression for the induced charge on the conductor

$$\sigma = \varepsilon_0 E_n = -\varepsilon_0 \frac{\partial V}{\partial n},$$

where $\partial V / \partial n$ is the partial derivative of the potential, V, with respect to axis normal to the surface. (If, for example, the positive [negative] x-axis were perpendicular to the surface, this would be the [negative of the] partial with respect to x.)

FARADAY CAGES AND SHIELDING

One of the most interesting applications of Gauss' law has to do with the electric field inside a charged sphere, which Gauss' law tells us should be equal to zero. In fact we may do Problem 11 on the board and compare it to the much more difficult mathematical effort we needed to do Problem 7, which shows us the same results (this should serve as an advertisement for how useful Gauss' law is in allowing us to actually avoid doing any heavy lifting, integrationwise).

One cool application of this is that if you wrap of aluminum foil around a portable radio, you should no longer be able to receive a signal. Try it. This should work because the signals that are received by the antenna are simply electromagnetic waves. (The only thing that might gum up this prediction of mine is that these electric fields which are part of the electromagnetic waves are oscillating at about 100 MHz for FM signals, which makes them not quite electro*static*.) The foil is acting as a "Faraday cage", a metal enclosure that isolates its interior from whatever electric fields are outside. Similar examples would be the chassis of a car or airplane during a thunderstorm (another reason not to fly in plastic aircraft) or the metal mesh inside the window of your microwave oven. Here's another:

Faraday cage
http://www.youtube.com/watch?v=bZwlD-Z0zmE

[Omit the rest of Section 2.5.3: The Force on a Conductor. We will not bother with it.]

CAPACITORS, Section 2.5.4

Consider two conductors, with a charge of +*Q* on one, and a charge of -*Q* on the other. These charges will result in there being an electric field between the two conductors. There will also be a potential difference between them. It is much easier to describe them in terms of this voltage than the *E*-field, because it is a single scalar value, while the field is a vector which varies throughout space. Let's consider this potential difference. If we double the charge on either plate from +*Q* and -*Q* to +*2Q* and -*2Q*, the electric field will double as well, and since the potential difference is related to the electric field by $V = -\int_0^{\vec{r}} \vec{E} \cdot d\vec{l}$, the potential difference will double as well. In other words, the voltage between the two conductors is proportional to the charge. In other*, other* words, the ratio of charge to voltage, *Q/V*, is a constant for any particular set of two conductors. We call this ratio the capacitance, *C*:

$$C = Q/V$$

with units of Farads (*F=C/V*). Careful: *V* stands for electric potential ("voltage") and Volts, respectively, in the definition of capacitance above and the definition of Farads to the left, while *C* stands for capacitance and Coulombs (which are *not* interchangeable), respectively, in the same two equations.

There are a few systems of conductors ("geometries") that come up again and again, so we should document how capacitance is related to the dimensions of the capacitor in each case. Also, these systems will give us some practice (a) in calculating the electric field using Gauss' law, and (b) in calculating the potential difference by line integration. A little practice is probably a good thing, no?

PARALLEL PLATE CAPACITORS

The granddaddy of all capacitors is the parallel-plate capacitor. This is simply two similarly-shaped flat sheets of metal, with areas *A*, placed a distance *d* apart. If $A >> d^2$, then we can approximate the electric field between the two plates by the field between two infinite sheets having uniform surface charge distributions. Remember from our section on Gauss' law examples that the field is uniform on either side of such a sheet, with $E=2\pi k\sigma$, where σ is the charge per area on the sheet. If *two* such sheets are placed near each other, their fields cancel out outside their "sandwich", but add inside. To get the voltage between the two sheets, note that the integral we just mentioned reduces to *V=Ed*. And so,

$$C = \frac{\varepsilon_0 A}{d} = \frac{A}{4\pi kd}$$

Below are two videos, one which shows how the capacitance of such a capacitor varies with plate spacing; the other one shows a charged object levitating above the bottom plate of such a capacitor.


Variable-thickness parallel plate capacitor
http://www.youtube.com/watch?v=JyKIenmfUHI


Capacitor levitation
http://www.youtube.com/watch?v=RqgslPPHvDw

ISOLATED SPHERE CAPACITORS

A more difficult system to visualize as a capacitor is a single spherical conductor. Think of this as a two-conductor system with one conductor an infinite distance away. Even if it were not at infinity, if it were at a distance much larger than the radius of the sphere, it is a useful approximation to imagine it infinitely far away. Using Gauss' law to get E, and integrating to get V, we find

$$C = 4\pi\varepsilon_0 R = R/k$$

You can consider your finger, reaching for a doorknob on a dry winter's day, as such a capacitor. Given that air breaks down (sparks) at about 3MV/m, you can now calculate the amount of charge built up in your finger if you can cause a spark to fly 1 cm. (*V=Ed=30kV*, assume *r=5mm*, which gives you *C=0.55pF*, so *Q=CV=17nC*.) This video shows what sort of mischief your finger-capacitor can get you into.


The hazards of static cling (see very end of clip)
http://www.youtube.com/watch?v=IdcPeW1XwKs

CONCENTRIC SPHERE CAPACITORS

Another good exercise is the case of two nested capcitors, one inside the other, both centered about the same point, with radii *a* and *b*, *b>a*. You should be able to show (thanks to Gauss and a line integration) that

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{ab}{k(b-a)}$$

Convince yourself that if *b>>a*, then this expression reduces to the case of the single spherical conductor above, with *d=b-a* and $A=4\pi a^2$.

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