WEEK 3: Go to class 5, 6

Homework assignments

CLASS 5:

The beginning of this lecture is is probably a good time to review what we have done in the last two weeks.

FORCE AND ENERGY

What are the two most important terms in all of Physics? For me those would be force and energy. Each of these words refers to a process for examining or interpreting phenomena in nature. The "force approach" involves applying Newton's second law to some object or system, resolving all of the forces into their components, and relating those to the acceleration of the object or system. The "energy approach" involves identifying all of the kinetic and potential energies in a system, and setting them constant if the total mechanical energy is conserved, or else setting the change in total mechanical energy equal to the work done by all nonconservative forces. The beauty of this is that the force approach and the energy approach should give you exactly the same results: neither system is better than the other. This means that you can choose which procedure to use. Which do you choose? Choose whichever one is easier. Usually, that means choosing the "energy approach" in order to avoid having to deal with vectors. In the next bit of this course, we will switch from using the force approach to using the energy approach in electrostatics.

SECTION 2.3

THE ELECTRIC POTENTIAL, *V*

In the last lesson we showed that that the electric field produces a conservative force, which means that we can define a potential energy associated with electric fields. Remember from mechanics that potential energy, *U*, and force are related by

$$\vec{F} = -\vec{\nabla} U$$

Therefore we would expect the electric potential energy of a system of two charges to be equal to

$$U = \frac{kqQ}{\mathscr{r}}$$

(Notice that this is a *scalar* equation.) Now, just as we found it necessary to introduce an electric field *E* in order to separate out the influence of the source charge from its effect on the test charge now, it is necessary to introduce a corresponding quantity such that, if we call this new quantity *V*,

V is to *U* as *E* is to *F* ("$V : U = \vec{E} : \vec{F}$ ")

$$\vec{E} = -\vec{\nabla} V$$

We call this quantity the electric potential. You are already familiar with this quantity from lab as "voltage", but voltage is always a *difference* in electric potential. Absolute potential has no physical meaning, just as the absolute height of an object has no physical meaning: we can define the zero height to be the level of the floor, the ground outside, sea level, or the center of the Earth. Voltage is always defined as the electrical potential *relative* to some convenient value, called the "ground" potential. The electric potential is independent of the test charge, Q, just as the electric field is, and so the electric potential due to a single source charge is

$$V = \frac{kq}{\mathscr{r}}$$

in units of Volt, *V=J/C*. Notice also that we can now rename the units of electric field, which were *N/C*, as *V/m*.

SUPERPOSITION

How does this simplify our lives? Because the electric potential, V, is a scalar, the contributions to the potential due to a collection of source charges simply add. No need to break V into its components: it has none. What about the potential energy of a collection of charges? Here you have to be careful not to do any "double accounting". Simply add up the potential energy components due to each pair of charges, excluding any "self energies" (those terms where *q* and *Q* are the same), and not counting any pair twice.

$$V = \sum \frac{kq_i}{\mathscr{r}_i}$$

$$U = \sum_{i \neq j} \frac{kq_i q_j}{\mathscr{r}_{i,j}}$$

INTEGRATION, Section 2.3.4

Just as for electric fields we need to consider what happens when we integrate over distribution of charges, we can calculate the electric potential due to a collection of charges by integrating over the charge, *dq*. Again, you will want to substitute and integral sign up for this summation sign, *dq* for *q*, and substitute one of the following for *dq*

$$dq = \begin{cases} \lambda\, dl \\ \sigma\, da \\ \rho\, d\tau \end{cases}$$

POISSON'S AND LAPLACE'S EQUATIONS, Section 2.3.3

Now it is time to play around with vector calculus again. We know the divergence of $\vec{E}$, and we know the relationship between $\vec{E}$ and *V*. Together, these two give us

$$\nabla^2 V = -4\pi k\rho = -\rho / \varepsilon_0$$

or Poisson's equation. In regions of space where there is no charge density, such as in the vacuum in deep space, the right hand side is zero, and we get

$$\nabla^2 V = 0$$

or Laplace's equation. We will spend a lot of time in Chapter 3 looking at ways of solving this equation. For most problems of interest to us, this special case (Laplace’s equation) is sufficient. Lucky thing, since it’s easier to solve.

CLASS 6:

SUMMARY SO FAR: Section 2.3.5

Study figure, 2.3.5 on page 87. This summarizes it all up so far: there are three principal quantities in studying electrostatics -- charge density *ρ*, electric field *E*, and potential *V*. Furthermore, if we know any one of these three quantities, we can calculate the other two. If I could wear any t-shirt to an exam, I would have this printed on a shirt and wear it. (Upside-down, of course, so that *I* can read it.

OK, it's very easy to get confused here because we have just in the last few weeks introduced four electrical quantities, all of which involve Coulomb's constant *k*, one or two charges *Q* or *q*, and a distance "*curly r*". Two of these quantities are vectors and two of them are scalars. Two of them are between two specific charges, two of them depend only on the source charge[s]. Here is a table that may help you keep them all straight:

NAME	SYMBOL	VECTOR/SCALAR	FORMULA	UNITS
Force	$\vec{F}$	Vector	$\vec{F} = \frac{kQq\hat{\mathscr{r}}}{\mathscr{r}^2}$	*N = kg·m/s²*
Electric field	$\vec{E}$	Vector	$\vec{E} = \frac{kq\hat{\mathscr{r}}}{\mathscr{r}^2}$	*N/C = V/m*
Potential energy	*U*	Scalar	$U = \frac{kqQ}{\mathscr{r}}$	*J = N·m*
Electric potential	*V*	Scalar	$V = \frac{kq}{\mathscr{r}}$	*V = J/C*

SECTION 2.4

WORK AND ENERGY IN ELECTROSTATICS

Now here's a philosophical question: Do electric fields actually exist? And how would we know?

We introduced electric fields as a sort of a shortcut, something that allowed us to imagine "someone else" producing the force between charges. Are these fields just an intellectual construction, or do they exist, and what properties would they have?

Well, imagine the following thought experiment. Consider a collection of charges in some region of space which produce some electric field. Now take another charge, a "test" charge, conveniently starting out at infinity, and bring it into the middle of these "source" charges. If the potential at infinity due to the source charges was infinity, more likely than not, the electric potential at the final resting spot of the test charge is not. This means that either work is done on the the test charge by the electric field, or else somebody had to do work against the electric field to move it. In the second case, that work which is done on the test charge can be gotten back by allowing it to return to infinity. So something is storing energy. But who?

This seems like a good thing to blame on the electric field. After all, where could the individual charges store the energy? If we do the calculation, we find it that all of space which contains an electric field can be thought of as also storing energy with a density --- energy per volume --- of magnitude

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = E^2/8\pi k$$

There is actual energy stored in the space containing the E-field. Therefore, $\vec{E}$ must be a real entity.

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