WEEK 2: Go to class 3, 4

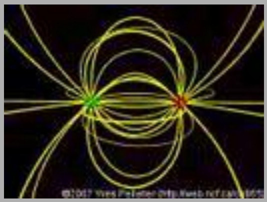
Homework assignments

CLASS 3:

ELECTRIC FIELD LINES, Section 2.2.1

Every textbook that covers electric fields gives at least lip service to the problem of drawing the electric field lines in the vicinity of a couple of source charges. Few of them do this well. Unfortunately we don't have a lot of time to do this well either. But I do want you to be aware of some of the properties of the electric field lines. First of all, the lines of electric field converge and diverge from a point charges. Second, the number of lines converging from or diverging from a single charge is proportional to the amount of that charge, with lines diverging from positive charges and converging on negative charges. Finally, the electric field in some position in space points in the direction tangential to the field lines at that point and the magnitude of the field at that point is proportional to the density of lines at that place (in other words the number of field lines per area perpendicular to the lines).

Luckily there are some good animations to help make sense of electric field lines. (A tip of the hat to the Physics Dept. at Coastal Carolina University) Try animations #1 and 2 before class. Also, here's a video to help you visualize the 3-dimensional nature of these lines:



E-field of dipole

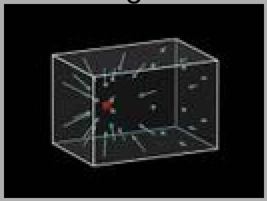
http://www.youtube.com/watch?v=bG9XSY8i_q8&feature=related

Since the number of lines that you decide to draw emerging from say a 1nC charge is arbitrary, you might be surprised to know that there is an exact physical analog to the number of electric field lines. It is called the electric flux, Φ_E.

$$\Phi_E = \int \vec{E} \cdot d\vec{a}$$

If we work backwards from the expression above, and differentiate both sides, we get that the electric field is equal to $d\Phi/dA$, that is, the electric field is equal to the density of flux per area. Since up above we said that the electric field is proportional to the number field lines per area, this means that the flux is directly proportional to the number of field lines. I can say that they are equal, because the number of field lines is arbitrary and depends on the scale that the person drawing the lines of field chooses. (It turns out that for magnetic field lines, flux is quantized, and so one can actually talk about an unambiguous and nonarbitrary number of field lines which doesn't depend on the artist's choice.)

Electric flux is really useful for only one thing in electrostatics. And that is Gauss' Law. Notice that I said that the electric field lines begin and end (converge or diverge) only on charges. That means that if we surround a collection of charges with some closed surface, then the flux through that surface will not change if we distort the shape of the surface in any way that does not increase or decrease the number of charges inside. In fact it is only the charges inside that count.



Electric flux through a box

One can show rigorously and mathematically that

$\oint \vec{E}\cdot d\vec{a} = 4\pi k Q_{enclosed}$, which the book writes as $\oint \vec{E}\cdot d\vec{a} = Q_{enclosed}/\varepsilon_0$

Unfortunately, trying to use this equation to calculate the electric field may seem about as useful as trying to look up in the dictionary how to spell word that you don't know how to spell: how you find the word in the dictionary in the first place? In this equation, we have the quantity that we wish to calculate, the electric field, inside the integral on the lefthand side. What we really want is an equation with the electric field on one side of the equation and all that other stuff on the other side. But Gauss' Law really is a useful equation. Why? For certain problems, (And here's the fine print: it only works for *certain* relatively *symmetric* problems.) you can make simplifications on the left-hand side that allow you to figure out the electric field based on the symmetry.

TRICKS FOR USING GAUSS' LAW

The most important question is how do you choose a Gaussian surface -- the surface over which the integral is evaluated and the surface inside of which the "enclosed charge" resides? The book doesn't do a wonderful job of walking you through this, and so I will try to state the steps more explicitly. The best way to learn, of course, is by doing, so we will do various problems in class.

1. Sketch the field lines in this problem, using what you know about the symmetry of the problem.
2. Choose surfaces either parallel to or perpendicular to the electric field lines.

If you follow these steps, the integral will be simplified into at most a few simpler integrals over portions of the surface. Over any parts of the surface for which the electric field lines are *parallel* to the surface, the contribution to the integral will be zero, because $\vec{E}\cdot d\vec{a}$ is zero. For those surfaces perpendicular to the electric field lines, the integral simplifies into a scalar integral of $\vec{E}\cdot d\vec{a}$. If, in addition, the electric field is uniform over that surface, as is the case for the simplest problems (most of which we will do in class) then we can further simplify by taking that constant *E* outside of the integral, leaving only the integral of *da* inside. The integral of da is simply the area of the surface, so the lefthand side of the equation is just *EA*.

APPLICATIONS OF GAUSS' LAW, Section 2.2.3

That's the drill. When you can apply Gauss' law (and you can do so only in a handful of cases) it works like a charm. What we will do in class is to look at the most common cases in which you can apply Gauss' law. I will expect you to be able to recreate any of these kinds of problems, and I expect you to keep the results of these four general cases handy for future reference. These four cases are

<u>0 dimensions:</u>	a point charge:	E proportional to r^{-2}
<u>1 dimension:</u>	a linear charge density:	E proportional to r^{-1}
<u>2 dimensions:</u>	a surface charge density:	E proportional to r^0 = constant
<u>3 dimensions:</u>	a volume charge density:	E proportional to r^1 = r.

Notice also that the mathematics for the force between two charges in Coulomb's law is identical to the mathematics for the force between two masses in the universal law of gravitation. This means that if the earth were a hollow sphere then the gravitational force inside would be zero, ignoring for a moment the sun's gravitational pull. There is a whole literature of science fiction, including some of the work of Jules Verne (who ought to have known better) which postulates civilizations living inside a hollow earth. You now have enough mathematics at your disposal to show that all such stories are just hogwash. In class we will probably show how the electric field inside a charged spherical shell or the gravitational force inside a hollow massive shell is zero because the object experiences canceling forces from opposite directions of the shell. (See Prob. 2.8 and 2.12) We will probably also consider what happens if you fall through an elevator shaft drilled through an asteroid.

CLASS 4:

VECTOR CALCULUS: DIV, GRAD, CURL, LAPLACIAN, Section 1.2

All right, time to dig into our toolbox of vector calculus from the Math Methods course and/or Chapter 1 of this text. In today's lecture we will look at the del operator, as well as the basic operations involving it, namely gradient, divergence, curl, and the Laplacian. If you are not already familiar with all of these, perhaps the best way of thinking about them is to use a descriptive analogy for each. The del operator is defined as

$$\vec{\nabla} = \hat{i}\,\tfrac{\partial}{\partial x} + \hat{j}\,\tfrac{\partial}{\partial y} + \hat{k}\,\tfrac{\partial}{\partial z}$$

with its various operations:

Gradient:	$\vec{\nabla} f$	"steepness"
Divergence:	$\vec{\nabla} \cdot \vec{F}$	"extroversion"
Curl:	$\vec{\nabla} \times \vec{F}$	"vorticity"
Laplacian:	$\nabla^2 f$	"lumpiness"

OK, this should all be review, although we will do a few problems at the board to brush off your cobwebs. If it is not merereview, then you need to refresh your memory or catch up. Crack open Chapter 1 Section 2.

DIVERGENCE OF $\vec{E}$, Section 2.2.2:

Now, time to go to the more advanced vector calculus. The first bit for us to consider is what is known as Gauss' theorem.

$$\oint \vec{E} \cdot d\vec{a} = \int_{vol} \left(\vec{\nabla} \cdot \vec{E}\right) d\tau$$

Notice the similarity with Gauss' Law, which is a physical principle, not a mathematical theorem like this equation. By equating the right hand side of both Gauss' law (last lecture) and Gauss' theorem (above), we get the following expression for the divergence of the electric field:

$$\vec{\nabla} \cdot \vec{E} = 4\pi k \rho, \qquad \vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$$

This is equivalent to saying that the E-field lines converge on and diverge only on charges. Notice that the book prefers to write everything in terms of ε_0, whereas your instructor is probably more likely to write everything in terms of Coulombs constant, *k*. Make yourself comfortable with both ways of writing this.

CURL OF $\vec{E}$, Section 2.2.4:

The other important bit of vector integration is Stokes' theorem, which involves the curl of a vector function.

$$\int \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{s}$$

Notice that the lefthand side is an integral over some surface, and the righthand side is a [closed] line integral over the boundary of the same surface. Rather than using the book's approach to evaluating the curl of the electric field, $\vec{E}$, let me point out that the curl of the electric fields produced by a point charge is always equal to zero, so that if we have a electric field produced by a collection of point charges, then that electric field will always have curl of zero.

$$\vec{\nabla} \times \vec{E} = 0$$

If the curl of $\vec{E}$ is zero, then the curl of the force, $\vec{F}$, is also equal to zero. Looking back at Stokes' theorem, this means that the closed path integral (The righthand side of the equation at the start of the section) is equal to zero as well. Any force for which this integral is zero is called a “conservative force”. That closed integral is nothing more than the work done by that force in going through space along some path and returning to the same point from which it began. For such forces we can draw several conclusions:

1. The net work done by such a force in traveling a closed path is zero
2. The net work done by such a force in going from point A to Point B is independent of the path
3. We can define a potential energy for this force

(These principles should be familiar to anyone who sat through *Mechanics*.) Maybe we should consider what kind of forces are *not* conservative. The best example of this is the frictional force. (along with viscous forces and other dissipative forces) If you push a book around a table in a circle, the work, always opposing the direction of motion, does net negative work on the book as it completes the circle. Therefore, friction is a non-conservative force (as is the force with which you have been pushing the book). There is no potential energy function associated with it.