LECTURE NOTES FOR PHYSICS 307: CLASSICAL MECHANICS
TEXT: THORNTON AND MARION’S CLASSICAL MECHANICS OF PARTICLES AND SYSTEMS

SOME USEFUL REFERENCE STUFF: Greek alphabet, metric prefixes, conversion factors

ASIGNMENTS: (Subject to change: check back often.)
HW #6: Due Friday, Oct. 24: Problems 3.4b, 7, 10, 22.
HW #7: Due Friday, Oct. 31: Problems 4.4, 5, 6, plus spreadsheet exercises. (Download the spreadsheets ODEDrivenSHO.xls, ODECubicPendulum.xls, and ODEPlanarPendulum.xls. You will need to enable the macros.)

“Dailies”:
Tuesday, Oct. 21: Show that the KE is maximum when \( \omega = \omega_0 \).

Thursday, Oct. 30: Consider the solution \( x=0 \) to the logistic equation. Verify that this is a solution, and show for which values of alpha do nonzero values of \( x_n \) converge to zero. (For what values is the solution stable?)

WEEK 9: GO TO LECTURE 14, 15

LECTURE 14: SECTIONS 3.7-8 (Return to top.)

Now let us consider electrical analogues of the harmonic oscillator. (more about this in Physics 308, Electricity and Magnetism) Consider a circuit consisting of a resistor, inductor, and a capacitor -- an LCR circuit. if we apply a Kirchhoff’s voltage rule to the circuit (the sum of the voltage gains equals the sum of the voltage losses) we get a differential equation identical to the equation for a driven harmonic oscillator, except that the names of all of the quantities have been changed. We can recycle all of our discussion of damped and driven oscillators by simply making the following substitutions

<table>
<thead>
<tr>
<th>Mechanical quantity</th>
<th>Electrical quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>position, ( x )</td>
<td>charge, ( q )</td>
</tr>
<tr>
<td>velocity, ( v )</td>
<td>current, ( I )</td>
</tr>
<tr>
<td>mass, ( m )</td>
<td>inductance, ( L )</td>
</tr>
<tr>
<td>frictional constant, ( b )</td>
<td>resistance, ( R )</td>
</tr>
<tr>
<td>1/Hooke’s constant, ( 1/k )</td>
<td>capacitance, ( C )</td>
</tr>
<tr>
<td>force, ( F )</td>
<td>applied voltage, ( E )</td>
</tr>
</tbody>
</table>

THE PRINCIPLE OF SUPERPOSITION
One thing that may have been bothering you about all of our discussion up to this point is that we have considered only sine or cosine waves. What happens if you drive an oscillator with something else? What happens if you apply a square wave or triangular wave to an LCR circuit?

Not to worry. The reason is that if you apply a force or voltage which is periodic, Fourier’s Theorem says that we can rewrite periodic function as the sum of cosine and sine waves. All of this good stuff is in your Math Methods book. We will look at a couple of simple periodic functions, namely the square wave and the triangular wave, and how to create each of them using only sine and cosine waves.

LECTURE 15: SECTIONS 4.1-4 (Return to top.)

CHAPTER 4: NONLINEAR OSCILLATIONS AND CHAOS
In the last chapter, we made an approximation that allowed us to simplify things. The potential energy for a pendulum is

\[ U = mgl(1 - \cos \theta), \]

and so the force is

\[ F = -mgl \sin \theta \approx -mgl\theta \]
The approximation we made was to say that $\sin \theta$ is practically equal to $\theta$. This simplifies things because it allows us to use Hooke's law to solve the motion of a pendulum. The result is that it behaves just like a mass on a spring. But we lied. $\sin \theta$ can be expressed as a power law that begins with the term $\theta$, but it has higher-order power terms. For small angles this is not a problem, but small angles are boring. Let's look at what happens for larger angles.

Any object that oscillates obeys Hooke's law for small enough oscillations. But, in the general case, there will be higher-order terms. If we set $x=0$ as the equilibrium point, then, for motion about that point, these higher order terms must all be of odd powers of $x$ for the force, and even for the potential energy. That is,

$$F = -kx + \varepsilon x^3 \quad \text{and} \quad U = \frac{1}{2}kx^2 - \frac{1}{4}\varepsilon x^4$$

These more general cases are known as nonlinear oscillators. There are two types of nonlinear oscillators, "hard" and "soft". The book shows graphically what these two cases look like, but an easier way to remember them is to think of the correction to the "hard" oscillator producing a larger force while the correction to the "soft" oscillator producing less of a force.

But why are these nonlinear cases of interest? We will look at a special case of nonlinear oscillation, the cubic oscillator, in which the force is proportional to the third power of the displacement. Play around with the ODECubicPendulum.xls simulation. If you set $\omega=0.01$, $\beta=0.03$, $x_0=10$, $v_0=0$, and $dt=0.03$, you can spot some of the differences between this system and the simple harmonic oscillator. We'll look at this specific case in class.

THE PLANE PENDULUM

This is where those phase diagrams come in handy. A linear oscillator produces a elliptical phase diagram. What will a nonlinear oscillator produce? Let's began with the pendulum and consider three limiting cases: the small angle case, the case in which the pendulum bob travels around its pivot in a vertical circle, and the bordering case where it is on the verge of circling.

For the last two cases, it is necessary to solve the integral that is derived in equation 4.27 and "prettified" in equation 4.28: an elliptic integral. Run away from such integrals whenever possible. We can solve for an exact solution only in the limiting case, in which case we get a definite integral which goes to infinity. (not very useful) In the small angle limit, we can expand this to get an expression for the "correction" to the linear expression (last chapter's expression) for the period of a pendulum. That is,

$$T = 2\pi \sqrt{\frac{g}{l}} \left[ 1 + \frac{k^2}{4} + \ldots \right]$$

where $k = \sin \frac{\theta_0}{2}$ and $\theta_0$ is the amplitude of the pendulum. This expression is useful for determining what the limits of the small angle approximation. That is, when can we ignore nonlinear effects. The other two cases are best visualized with phase diagrams.

To see what effect varying the angle has on the period, you can check out the following simulation. (Note that what the simulation calls the "theoretical" period is just the small-angle limit. For $\theta=179$ degrees, the actual period is about four times larger.)