THE STEFAN-BOLTZMANN LAW

Electromagnetic radiation absorbed and emitted by any substance is dependent on the temperature of the substance. Josef Stefan showed experimentally in 1879 that for a perfect emitter (a 'black body') the rate at which energy is emitted is related to the object's temperature by

$$P = \sigma \varepsilon A T^4, \qquad [1]$$

where ε equals *1* for a perfect blackbody, *A* is the surface area of the radiator, and σ is known as the Stefan-Boltzmann constant. This relation was subsequently derived theoretically by Ludwig Boltzmann, who had been a student assistant of Stefan's at the University of Vienna, and later succeeded to Stefan's professorial post there. In 1900 Max Planck showed that the spectral distribution of energy density in a blackbody could be described with what we call the Planck radiation formula:

$$\bar{u}(f) = \frac{8\pi h}{c^3} \frac{f^3 df}{e^{hf/kT} - 1} \qquad [2]$$

This expression required the use of the quantum hypothesis and marked the first appearance of the quantum of energy, *hf*. If we integrate $\bar{u}(f)df$ over the range of frequencies from 0 to ∞ we should get the total energy density (per volume), U, which is related to the emissivity,*E=P/A*, by *E=cU/4*.

LIBRARY RESEARCH:

Find out more about the following: blackbodies, Stefan-Boltzmann's (T^4) law, Wien's law.

THEORY:

Integrate the expression in Eq. 2, and multiply by c/4 to get the Stefan-Boltzmann law (Eq. 1), for an emissivity of 1. Verify that the constants multiplying your factor of T^4equal the accepted value of the Stefan-Boltzmann constant, σ. (Hint: use the dimensionless quantity *x=hf/kT* to simplify the integral. *Look up* the value of the resulting*definite* integral in *x*. (HINT: do not try to solve it analytically.))

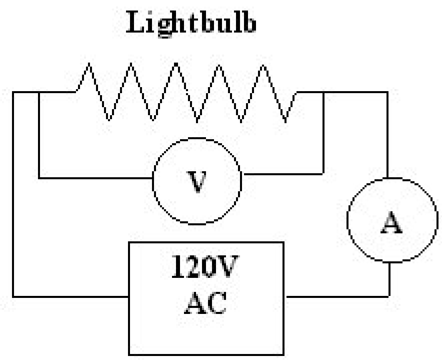
EXPERIMENT:

You will use a radiation sensor to measure the intensity of thermal radiation. The sensing element, a miniature thermopile, produces a voltage proportional to the intensity of the radiation. (Look on the sensor to find out the conversion between its reading and power radiation.) When using the sensor, be careful not to change its position. Instead of opening and closing the aperture for every measurement, leave the aperture open and place a piece of insulating foam between the source and the sensor. Make measurements quickly so that the sensor does not heat up when exposed to the source.

You will be measuring the radiation emitted by two sources. The 'high temperature' source is a tungsten-filament lamp. The temperature of the filament can be calibrated to the resistance of the filament using an optical pyrometer. The 'low temperature' source is a thermal radiation cube a "Leslie cube"). This is a cube of nearly uniform external temperature, heated from inside by a light bulb. The sides of the cube are black, white, shiny metallic, and dull metallic ("gray").

A. STEFAN-BOLTZMANN LAW AT HIGH TEMPERATURES.

Set up the circuit as shown. In order to determine the resistance of the filament and the power dissipated at various temperatures, you will record *V* and *I* at different voltage levels.



Before you do that, though, measure the room-temperature resistance of the bulb to three digits ***using a four-wire ohmmeter***. (Ask your instructor to show you how.) We will use this quantity, called R_{300K} later in our calculations. Measure this resistance with enough precision that you can see the resistance change as you warm the bulb in your hand. Starting at *half* the voltage of where the filament first begins to glow, record V and I every 5V up to about 110V.

Record V and I at each voltage level on a spreadsheet. Calculate R, R/R_{300K}, and temperature, using the tungsten resistivity data table below, and $P=VI$ at each voltage. Plot the power you have measured vs T^4 (Use *Kelvin* temperature values!!). Do your data support the Stefan-Boltzmann law?

TUNGSTEN RESISTANCE VS TEMPERATURE

R (Ohm)	R/R_{300}	Temperature (K)	R (Ohm)	R/R_{300}	Temperature (K)	R (Ohm)	R/R_{300}	Temperature (K)
	1.00	300		7.14	1500			
	1.43	400		7.71	1600		14.34	2700
	1.87	500		8.28	1700		14.99	2800
	2.34	600		8.86	1800		15.63	2900
	2.85	700		9.44	1900		16.29	3000
	3.36	800		10.03	2000		16.95	3100
	3.88	900		10.63	2100		17.62	3200
	4.41	1000		11.24	2200		18.28	3300
	4.95	1100		11.84	2300		18.97	3400
	5.48	1200		12.46	2400		19.66	3500
	6.03	1300		13.08	2500		20.35	3600
	6.58	1400		13.72	2600			

B. STEFAN-BOLTZMANN LAW AT LOW TEMPERATURES.

With the thermal radiation cube set to its highest setting, measure the power emitted by each of its surfaces as a function of temperature for the entire temperature range of the cube. (Be sure to have the sensors the same distance from each surface.) You will use the internal thermistor temperature sensor of the cube, the resistance of which changes with temperature. The calibration is on the side of the cube. Measure the thermistor's temperature at each temperature!! Do all the surfaces emit with the same power rate? (Why or why not?) Remember that at low temperatures, energy loss to the surroundings is significant. The net energy transfer goes as $T^4 - T_o^4$, where T_o is room temperature. (You should be able to explain *why* that is.) Use *Kelvin* temperature values!!

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