

HOOKE'S LAW AND A SIMPLE SPRING

DONALD C. PECKHAM

PHYSICS 307

FALL 1983

(Digitized and Revised, Fall 2005)

ABSTRACT

The spring constant of a screen-door spring was determined both statically, by measuring its elongation when subjected to loading, and dynamically, by measuring the period of a mass hung from one end and set into vertical oscillation. The resulting values of 218.6 ± 0.4 N/m and 217.8 ± 1.0 N/m, respectively, indicate that the spring's behavior follows Hooke's law to within the limits of accuracy of the experiment.

A formal lab report should include a title page like this one, with all of the appropriate information -- a descriptive title, your name, the course title, the date, and an abstract.

The abstract must be able to stand by itself, it must be brief, and it must include the principal numerical results of the experiment. Its purpose is to help a reader to decide whether your report is of particular significance or interest for the reader to continue. Generally its structure consists of three parts: you need to describe...

- *What did you do,*
- *What were your results,*
- *What do these results tell you.*

You will discuss these three items, respectively, in greater detail later in your report in

- *the Procedure,*
- *the Data and Conclusions sections,*
- *the Discussion section.*

The report which follows should generally be written double-spaced, preferably single-sided, unlike this example.

INTRODUCTION

If it is not stretched to the point where it becomes permanently deformed, the behavior of a properly wound coiled spring, when subjected to a stretching force, can be expected to follow Hooke's Law. [Bueche, p. 95] (Note that Hooke's Law applies more generally to many more systems than just ordinary springs.) To see whether an ordinary screen door spring behaves similarly, one such spring was suspended by one end from a horizontal support and masses were hung from its other end to stretch it as shown in Figure 1. The resulting data were used to construct a graph of load as a function of elongation, from which it was possible to obtain the spring constant of the spring. In addition, for one value of load the spring was given a small additional stretch and released, thereby setting the system into vertical oscillation. Assuming this motion to be simple harmonic, its period also yields a spring constant, thereby providing an additional check.

Note the difference between the abstract and the introduction. The introduction indicates the primary thrust of the experiment but does not indicate results. It does not have to stand by itself and can refer to later parts of the report such as a diagram or a graph. Such reference should be to a numbered figure or graph. (Figure 1 in this example)

THEORY

It is essential that the theory section be written for the particular audience for which the report is intended. In this case, imagine your audience to consist of one of your classmates who had to miss this week's lab.

If a weight, $W = mg$, is hung from one end of an ordinary spring, causing it to stretch a distance x , then an equal and opposite force, F , is created in the spring which acts to oppose the pull of the weight. If W is not so large as to permanently distort the spring, then this force, F , will restore the spring to its original length after the load is removed. F is thus called an elastic force and it is well known that the magnitude of an elastic restoring force is directly proportional to the stretch,

$$F = kx$$

Note how all quantities are defined when they first appear. Notice how the author uses italics to set off physical quantities from the rest of the text.

A relationship called Hooke's Law after the 17th century scientist who studied it. The constant k is called the spring constant, or stiffness coefficient. To emphasize that x refers to the change in length of the spring we write

$$F = mg = k\Delta\ell. \quad (1)$$

In this form it is apparent that if a plot of F as a function of $\Delta\ell$ has a linear portion, this provides confirmation that the spring follows Hooke's Law [Bueche, p. 226] and enables us to find k .

An additional approach is possible. One definition of simple harmonic motion (SHM) is that it is motion under a linear, "Hooke's Law" restoring force. For such a motion we have, from Newton's second law,

$$F = -kx = ma.$$

The minus sign appears since in this case the acceleration of the object in SHM is in the direction opposite to the force causing it. From this definition

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Write your equations in a form that is easy to read. In this case, Equation Editor does the job. In Microsoft Word, choose the Insert menu, then the Object menu, then select Equation Editor.

The solution to this differential equation is given in standard physics textbooks [Bueche, p. 231] and the resulting expression for the period, T , of the SHM is

Since your reader can be assumed to have seen this derivation previously, it is not necessary for you to repeat it.

$$T = 2\pi\sqrt{m/k} \quad (2)$$

where k is again the spring constant and m is the mass which is under motion. This provides an additional method for testing whether the spring obeys Hooke's Law.

PROCEDURE

The Procedure elaborates on the first item of the Abstract: 'what did you do?' Note that the procedure is not a step-by-step set of instructions but is meant to point out unusual ways of doing things or special items which another investigator would need to know in order to repeat your work.

We hung a common screen door spring directly from a horizontal metal rod secured to the laboratory bench by a C-clamp, another pole and a right-angle clamp. We attached a mass hanger directly to the bottom of the hanging spring to enable us to vary the load hanging from the spring. Vernier calipers were used to measure the vertical distances, l_0 and l_1 , (Fig. 1) from the horizontal support rod to the top coil and the bottom coil, respectively, of the hanging spring, for a variety of load masses ranging up to 9kg. (Table 1)

Masses were added to the spring in a more or less random order so as to better observe the overall correlation of load and elongation as we graphed the data during collection.

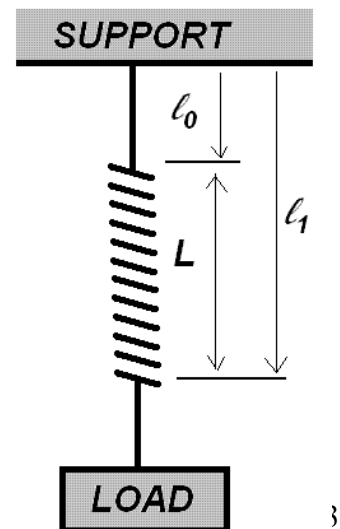


Figure 1. Spring and load

For ease in reading, the data are reproduced in Table 1 in order of increasing load. Mass, in kg, is plotted against elongation, in cm, in the graph in Figure 2.

For the SHM part of the experiment, a single mass of 4kg was hung from the spring and the time required for the system of mass plus spring to execute an integer N number of oscillations was measured with a digital stopwatch. This was repeated for three arbitrary values of N , and the data appear in Table 2.

Sketches and diagrams need not be works of art but they must clearly indicate the features they are meant to illustrate. All relevant quantities referred to in the text of your report must be clearly displayed on the diagram.

DATA AND CALCULATIONS

The Data and Calculations section elaborates on the second part of the Abstract: 'what were your results?'

Collect all of your data in one place. Include in the same tables derived values making sure you indicate the source of these derived values. This may require a sample calculation or a formula. Units are also essential.

The measured positions of the top (l_0) and the bottom (l_1) coils of the spring are given below in Table 1. From these distances we calculated the length of the spring ($L=l_1-l_0$), and the extension, ΔL , which is the difference between this extension, ΔL , and the unloaded extension of 38.8cm.

In Fig. 2 we calculate the spring constant from measuring 25 to 50 separate oscillations and calculating the period of oscillation, T . From the average value of the period, we can use Equation 2 to calculate the spring constant, k .

The mass used in calculating k was not merely the load attached to the bottom of the spring. The reason for this is that the system that is vibrating includes the spring itself. However, the entire spring does not vibrate with the same amplitude as the load and therefore it is reasonable to assume that the effective load is the mass hung from the end of the spring plus some fraction of the mass of the spring. The fraction used was 1/3 since similar experiments with other types of springs have led to this empirical result. [Sears, Zemansky, and Young] It was assumed that 1/3 of the mass was a reasonable approximation to the correct value for this particular spring.

Load	l_0	l_1	$L=(l_1-l_0)$	$\langle L \rangle$	ΔL
kg	cm	cm	cm	cm	m
0	31.9	70.7	38.8	--	
0	32.1	70.9	38.8	38.8	0
1	32.0	71.1	39.1	39.1	.003
2	32.1	74.3	42.2	--	
2	32.3	74.6	42.3	42.25	.034
3	32.1	79.1	47.0	47.0	.082
4	32.3	83.6	51.3	51.3	.125

5	32.3	88.1	55.8	--	
5	34.4	88.3	55.9	55.85	.171
6	32.4	92.6	60.2	--	
6	32.6	92.8	60.2	60.2	.214
7	32.5	97.2	64.7	64.7	.259
8	32.4	101.9	69.5	69.5	.307
9	32.5	106.5	74.0	--	
9	32.6	106.5	73.9	73.95	.351

Mass of spring 70 gm
 All position measurements $\pm .05$ mm
 All masses ± 3 gm

Table 1. Calculation of the spring constant from its loaded extension.

Oscillations	Time (sec)	T (sec)
50	42.80	.856
25	21.34	.854
40	34.11	.853

$\langle T \rangle = .854 \pm .002$ sec.

Timing oscillator - load = 4 kg

Table 2. Calculation of the spring constant using the resonant frequency approach.

Notice that uncertainties are also calculated. For the data in Table 2, the uncertainty used is just the standard deviation of the collection of calculations for T. For the data in Table 1, you should use a computer program that calculates the standard deviation of the slope and the intercept of the straight-line fit we see in Figure 2 below.

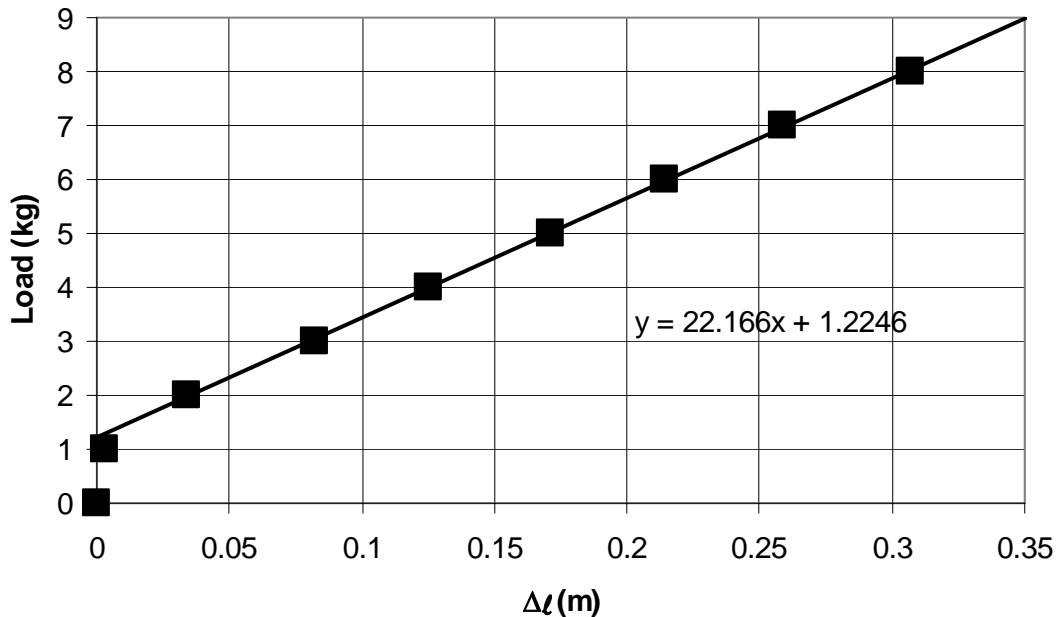


Figure 2. Load applied as a function of the resulting elongation of a screen door spring.

Graphs must conform to all the rules given in the General Lab Instructions. All graphs and figures should be numbered and referred to in the text by their number. This graph was inserted into this Word document by a simple Copy and Paste from Excel into Word. Graphs from other

sources may be physically cut and pasted into your report, if necessary, or converted into graphical files first.

DISCUSSION

The Discussion elaborates on the third part of the Abstract: 'what do these results tell you?'

Figure 2 indicates that for loads greater than about 1.1kg (notice intercept of fit), there is a linear relation between load and elongation but that for small loads such a relationship fails, since the fit curve does not intercept the y axis at zero. It is assumed that this is caused by an initial "set" in the spring which requires some initial load to overcome. This is apparent if one stretches the spring manually and then releases it. It seems to snap shut at the last moment. For this reason, 0 and 1 kg were ignored and the rest of the data were treated by a least squares analysis to determine the coefficients of first degree polynomial best fit. These were used to plot the line on the graph. The slope of the line, ignoring loads of less than 2kg, was found to be 22.2 ± 0.2 kg/m. From Equation 1, we see that we need to multiply this quantity by g to calculate a value for the spring constant of $k = 217.4 \pm 1.8$ N/m.

Note that final numerical values include an estimate of uncertainty.

In determining k dynamically, only one value of load (4.0 kg) was used. Since the mass of the spring was 7.0 gm the value of m used to determine k from equation (2) was 4.023 kg. This together with $\langle T \rangle = .854 \pm .002$ sec gives

$$k = \frac{4\pi^2 m}{T^2} = 217.7 \pm 1.0 \text{ N/m}$$

These two values of k show agreement within the limits of accuracy of the experiment and as such are evidence that the spring used, at least over a range of loads from 2 to 9 kg, obeys Hooke's Law.

If the dynamic experiment were extended by using a range of masses and a graph of T^2 as a function of m were constructed, a linear plot should result. For this graph

$$T^2 = \frac{(2\pi)^2}{k}(m + m_o) ,$$

where m_o would be the effective mass of the spring. This would give a more reliable result than the estimate of 1/3 of the spring mass used here.

BIBLIOGRAPHY

Bueche, F. J., Introduction to Physics for Scientists and Engineers, Third Edition, McGraw-Hill, N.Y. (1980).

Wilchinsky, Z., "Theoretical Treatment of Hooke's Law," Am. J. Phys. 7, 134 (1939).

Sears, F. W., Zermansky, M. and Young, H. D., University Physics, 5th Edition, Addison-Wesley, N. Y. (1981), as cited in Yost, S. A., "The effect of spring mass on the oscillation frequency", <http://homework.phys.utk.edu/courses/spring2002/phys221/spring.pdf>.

Peckham, D., "Hooke's Law: St. Lawrence University Physics Dept. Phys 307 Laboratory Instructions", Canton, N. Y. (1983), [it.stlawu.edu/~physics/InsertMadeupURLhere.html]

For reports in this course your bibliography should contain at least two items. They must be listed in a standard bibliographic form. (See, for example, Hacker's A Writer's Reference.)

Both formal and informal lab reports are subject to all the rules governing academic honesty, including issues of plagiarism, citation, and presenting others' work as your own. You are expected to be familiar with the rules, and your work will be considered with those rules in mind.