Driven Damped Harmonic Oscillations

EQUIPMENT

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INTRODUCTION

The oscillator consists of an aluminum disk with a pulley that has a string wrapped around it to two springs. The angular positions and velocities of the disk and the driver are recorded as a function of time using two Rotary Motion Sensors. The amplitude of the oscillation is plotted versus the driving frequency for different amounts of magnetic damping. Increased damping is provided by moving an adjustable magnet closer to the aluminum disk.

THEORY

The oscillating system in this experiment consists of a disk connected to two springs. A string connecting the two springs is wrapped around the disk so the disk oscillates back and forth. This is like a torsion pendulum. The period of a torsion pendulum is given by

\[ T = 2\pi \sqrt{\frac{I}{\kappa}} \]  

(1)

where I is the rotational inertia of the disk and \( \kappa \) is the effective torsional spring constant of the springs. The rotational inertia of the disk is found by measuring the disk mass (M) and the disk radius (R). For a disk, oscillating about the perpendicular axis through its center, the rotational inertia is given by \( I = \frac{1}{2}MR^2 \).
The torsional spring constant is determined by applying a known torque ($\tau = rF$) to the disk and measuring the resulting angle ($\Theta$) through which the disk turns. Then the spring constant is given by $K = \frac{\tau}{\Theta}$.

(2)

When the damped oscillator is driven with a sinusoidal torque, the differential equation describing its motion is

$$I \frac{d^2\Theta}{dt^2} + b \frac{d\Theta}{dt} + \kappa \Theta = \tau_o \cos \omega t$$

The solution to this equation is

$$\Theta = \frac{\Theta_0}{\sqrt{\left(\omega^2 - \omega_o^2\right) + 4\omega^2 b^2}} \cos(\omega t - \delta)$$

(3)

where $\delta = \tan^{-1}\left(\frac{2\omega b}{\omega_o^2 - \omega^2}\right)$ is the phase difference between the driving torque and the resultant motion.

(i) As the driving frequency ($\omega$) approaches zero, $\delta = \tan^{-1}(0) \rightarrow 0$. The resulting motion is in phase with the driving torque.

(ii) At resonance, $\omega = \omega_o$, which results in

$$\Theta = \frac{\Theta_0}{2\omega_o b} \cos \left(\omega_o t - \frac{\pi}{2}\right) \quad \text{and} \quad \delta = \tan^{-1}\left(\frac{2\omega_o b}{\omega_o^2 - \omega^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

(iii) As the driving frequency ($\omega$) goes to infinity, $\delta = \lim_{\omega \to \infty} \left[\tan^{-1}\left(\frac{2\omega b}{\omega_o^2 - \omega^2}\right)\right] = \pi$. The resulting motion is $180^\circ$ out of phase with the driving torque.
SET UP

1. Mount the driver on a rod base as shown in Figure 2. Slide the first Rotary Motion Sensor onto the same rod as the driver. See Figure 3 for the orientation of the Rotary Motion Sensor.

![Figure 2: Driver](image)

![Figure 3: Complete Setup](image)

![Figure 4: String and Magnet](image)
2. On the driver, rotate the driver arm until it is vertically downward. Attach a string to the driver arm and thread the string through the string guide at the top end of the driver. Wrap the string completely around the Rotary Motion Sensor large pulley. Tie one end of one of the springs to the end of this string. Tie the end of the spring close to the Rotary Motion Sensor.

3. Use two vertical rods connected by a cross rod at the top for greater stability. See Figure 3.

4. Mount the second Rotary Motion Sensor on the cross rod.

5. Tie a short section of string (a few centimeters) to the leveling screw on the base. Tie one end of the second spring to this string.

6. Cut a string to a length of about 1.5 m. Wrap the string around the middle step of the second Rotary Motion Sensor twice. See Figure 4. Attach the disk to the Rotary Motion Sensor with the screw.

7. To complete the setup of the springs, thread each end of the string from the pulley through the ends of the springs and tie them off with about equal tension is each side: The disk should be able to rotate 180 degrees to either side without the springs hitting the Rotary Motion Sensor pulley.

8. Attach the magnetic drag accessory to the side of the Rotary Motion Sensor as shown in Figure 4. Adjust the screw that has the magnet so the magnet is about 1.0 cm from the disk.

9. Wire the driver circuit as shown in Figure 5. In this experiment, a ramped voltage is applied to the driver using the signal generator on the 750 interface. However, since the driver motor stalls out at low voltages and it is desired to get the maximum number of data points possible, it is necessary to have an offset voltage so the minimum voltage is about 1 V. This offset voltage is supplied by the DC power supply. Plug the driver into the DC power supply and attach the digital voltmeter across the power supply.

![Figure 5: Driver Wiring Diagram](image)
10. Plug the disk Rotary Motion Sensor into Channels 1 and 2 on the ScienceWorkshop 750 interface with the yellow plug in Channel 1. Plug the driver Rotary Motion Sensor into Channels 3 and 4 with the yellow plug in Channel 3. Plug the Power Amplifier into Channel A.

11. Open the DataStudio file called "Driven Harmonic".

PROCEDURE

1. Measure the resonant frequency. Leave the DC power supply turned off and click the signal generator off in DataStudio. Click on START, displace the disk, and let it oscillate. Click on STOP. Measure the period using the Smart cursor on the disk oscillation graph.

2. Click on START. Hang a hooked mass (20 g) on the top of one of the springs and measure the resulting angle through which the disk rotates. Click on STOP. Measure the radius of the middle step of the RMS pulley and calculate the torque caused by the weight of the 20 g mass. Calculate the torsional spring constant using Equation (2).

3. Remove the disk and measure the mass and radius of the disk. Calculate the rotational inertia of the disk.

4. Turn on the DC power supply and set the voltage on 1 V. Click on Auto on the signal generator in DataStudio. Click on START in DataStudio. Since the frequency of the signal generator ramp is set for 0.001 Hz, data collection will take 1000 seconds (18 minutes). Then click on STOP.

5. Adjust the magnetic damping screw to about 0.5 cm from the disk and repeat the data collection.

6. Adjust the magnetic damping screw to about 0.2 cm from the disk and repeat the data collection.

7. Adjust the magnetic damping screw to about 0.5 cm from the disk and repeat the data collection.

ANALYSIS

1. Using the torsional spring constant and the disk rotational inertia, calculate the theoretical period and the resonant frequency of the oscillator (ignoring friction).

2. Examine the resonance curves for different amounts of damping. How does increasing the damping affect the shape of the curve (the width, maximum amplitude, frequency of the maximum)?

3. Is the resonant frequency for the least amount damping the same as the theoretical
frequency? Calculate the percent difference.

4. Why is the resonance curve asymmetrical about the resonant frequency?

5. Examine the graphs of the driving oscillation versus time and the disk oscillation versus time. Measure the phase difference between these oscillations at high frequency (at the beginning of the time), resonance frequency (at the time when the disk oscillation is greatest), and at low frequency (at the end of the time). Do these phase differences agree with the theory?