

# Chapter 26: Relativity

Are the laws of physics the same in different reference frames?

## Reference Frames

All motion is measured relative to a frame of reference

- Inertial**
  - Velocity of reference frame is constant
- Non-Inertial**
  - Reference frame is accelerating

## Example of reference frames

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## Principle of Relativity

- The **Laws** of physics are the same in all **INERTIAL** frames
- (the values of quantities are not necessarily the same, but the Laws of physics are the same.)

Special Relativity  
Inertial Reference Frames

General Relativity  
Includes non-inertial frames

## Relative velocity

In reference frame  $S'$   
The laser appears to travel at  $v' = v_{laser} = v_L$        $v' = v_L = c$

In reference frame  $S$   
The laser appears to travel at  $v = v_{space-ship} + v_L$        $v = v_L + c$

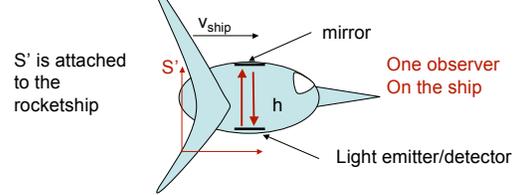
## Problem: speed of light can't be larger than $c$

- One possible solution: most waves travel in a medium. Water waves have water, sound waves have air. What if there were a medium for E-M radiation. We'll call it the "Ether". E-M would travel at  $c$  relative to the ether, and its speed could be larger or smaller relative to other reference frames.
- Michelson and Morely showed with their interferometer that there is no ether.
- Einstein solved the problem with his principles of relativity.

## Einstein's Postulates of Relativity

1. The laws of physics are the same in all **inertial** reference frames
2. The speed of light in vacuum is the same in all **inertial** reference frames

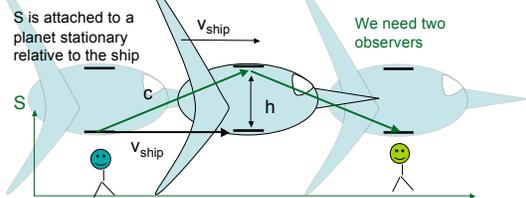
## Consequences of Einstein's postulates



$\Delta t'$  = Time to travel from emitter to mirror and back to the detector relative to  $S'$

$$\Delta t' = \frac{\text{distance}}{\text{speed}} = \frac{2h}{c}$$

## Consequences of Einstein's postulates



$\Delta t$  = Time to travel from emitter to mirror and back to the detector relative to  $S$

$$\Delta t = \frac{\text{distance}}{\text{speed}} = \frac{2h}{\sqrt{c^2 - v_{\text{ship}}^2}}$$

The two times are NOT the same

$$\Delta t' = \frac{\text{distance}}{\text{speed}} = \frac{2h}{c}$$

$$\Delta t = \frac{\text{distance}}{\text{speed}} = \frac{2h}{\sqrt{c^2 - v_{\text{ship}}^2}}$$

This disturbs our idea of absolute time

$$\Delta t' c = \Delta t \sqrt{c^2 - v^2}$$

$$\Delta t = \frac{\Delta t' c}{\sqrt{c^2 - v^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

Time in the **rocketship's Reference frame,  $\Delta t'$** , is **longer** than the time in the **planet's reference frame,  $\Delta t$** .

TIME IS RELATIVE

## Proper Time - time dilation

- We define "proper time",  $\Delta t_o$ , as the reference frame where the two events occur in the same place (only need one observer).

Which reference frame was that? **rocketship**

$$\Delta t = \gamma \Delta t_o$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$\gamma$  is always greater than 1  
 $v$  is the relative velocity between the two reference frames

## Example: Problem 26.5

- An unstable particle called the pion has a mean lifetime of 25 ns in its own rest frame. A beam of pions travels through the lab at a speed of 0.60  $c$ .
- What is the mean lifetime of the pions as measured in the lab frame?
- How far does a pion travel as measured by lab observers during this time.

Is length the same in different reference frames?

**S** is attached to a planet stationary relative to the ship

**S'** is attached to the rocketship and is moving at  $v_{\text{ship}}$

$L$  is the distance between ☺ and ☹ as measured in **S**.  $L'$  is the same distance measured in **S'**

Is length the same in different reference frames?

$L$  can be measured in **S** by having ☺ and ☹ measure time and taking the difference,  $\Delta t$ . Then  $L = v_{\text{ship}} \Delta t$

$L'$  can be measured in **S'** by having ☹ start the stop watch at ☺ and stop the stopwatch at ☹ to determine,  $\Delta t'$ . Then  $L' = v_{\text{ship}} \Delta t'$

Is length the same in different reference frames?

$L = v_{\text{ship}} \Delta t$

$L' = v_{\text{ship}} \Delta t'$

Which reference frame has proper time? **rocketship**

$L' = v_{\text{ship}} \Delta t_0$  or  $v_{\text{ship}} = L' / \Delta t_0 = L / \Delta t$   
 since  $\Delta t = \gamma \Delta t_0$ ,  $L' = L / \gamma$

### Proper Length

- We define proper length as the reference frame where the object or distance is at rest.
- Which reference frame has the proper length,  $L_0$ ? **Planet**

$L' = L_0 / \gamma$

So in general  $L = L_0 / \gamma$   
 Where  $L$  is any reference frame other than the one where the object is at rest.

### Proper Length example: 5.15

- A spaceship moves at a constant velocity of  $0.40c$  relative to an Earth observer. The pilot of the spaceship is holding a rod which he measures to be  $1.0\text{m}$  long. How long is the rod according to the Earth observer in the following situations?
- (a) The rod is held perpendicular to the direction of motion of the spaceship.
- (b) The rod is parallel to the motion.
- (c) If the rod is held at an angle of  $30^\circ$  above the direction of motion.

### Velocity transformations

- Galilean velocity addition formula

$$\mathbf{v}_{CA} = \mathbf{v}_{CB} + \mathbf{v}_{BA}$$

- Where  $\mathbf{v}_{BA}$  is the velocity of B with respect to A and  $\mathbf{v}_{BA} = -\mathbf{v}_{AB}$

## Velocity transformations

- Relativistic velocity addition formula
 
$$v_{CA} = \frac{v_{CB} + v_{BA}}{1 + v_{CB}v_{BA}/c^2}$$
- Where  $v_{BA}$  is the velocity of B with respect to A and  $v_{BA} = -v_{AB}$

## Example Problem 20

- Particle A is moving with a constant velocity of  $0.90c$  relative to Earth. Particle B moves in the opposite direction with a constant velocity of  $0.90c$  relative to the same observer on earth. What is the velocity of particle B as seen by particle A?

## Relativistic Momentum

- Our classic definitions of momentum ( $\mathbf{p} = m\mathbf{v}$ ) and kinetic energy ( $mv^2/2$ ) are fine as long as  $v \ll c$
- Relativistically
  - $\mathbf{p} = \gamma m\mathbf{v}$

## Momentum Graph

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- At what speed do the two definitions diverge?
- The momentum can increase without bound while the speed never reaches  $c$ .
- $\mathbf{F} = \Delta\mathbf{p}/\Delta t$  but  $\mathbf{F} \neq m\mathbf{a}$
- The acceleration due to a constant net force decreases as  $v \rightarrow c$

## Momentum Example

- Find the momentum of a lambda particle traveling at  $0.85c$ .  $m_\lambda = 1115.7 \text{ MeV}/c^2$
- Compare the relativistic and nonrelativistic momentum.

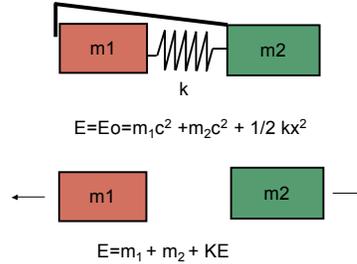
## Mass and Energy

- In general in our daily experiences, mass does not change when objects interact. So to look at the Energy due to mass would be like adding a constant value and would not change the results. Even in chemical reactions, it seems as if mass is conserved.
- However, we can turn mass into energy and vice versa!
  - Pair annihilation
  - Pair production
  - radioactivity

## Rest Energy

- Relativity requires that we consider the energy due to mass. Mass is not conserved, and it can change in an interaction. Energy is conserved, and mass is form of energy.
- Rest mass energy  $=E_o=mc^2$
- Units electron-volt  
 $1eV=1e \times V = 1.6 \times 10^{-19}J$

## Rest Energy



## Energy released in Beta decay: Example 26.5

- Carbon dating is based on the radioactive decay of a carbon 14 nucleus (6 protons, 8 neutrons) into a nitrogen 14 nucleus (7 protons, 7 neutrons) an electron and an antineutrino.  $^{14}C \rightarrow ^{14}N + e^- + \bar{\nu}$
- Find the energy released in this reaction.
- Masses  $^{14}C = 13.999950 \text{ u}$ ,  $^{14}N = 13.999234 \text{ u}$   
 $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

## Kinetic Energy

$$\text{Kinetic Energy} = K = (\gamma - 1)mc^2$$

$$\text{Total Energy} = E = K + mc^2$$

$$= K + E_o = \gamma mc^2$$

Total Energy is always conserved

## KE example

- If the kinetic energy of an electron is 200 keV, and it's rest mass is  $0.511 \text{ MeV}/c^2$ , what is the speed of the electron? What is its total energy?