Projectile Motion

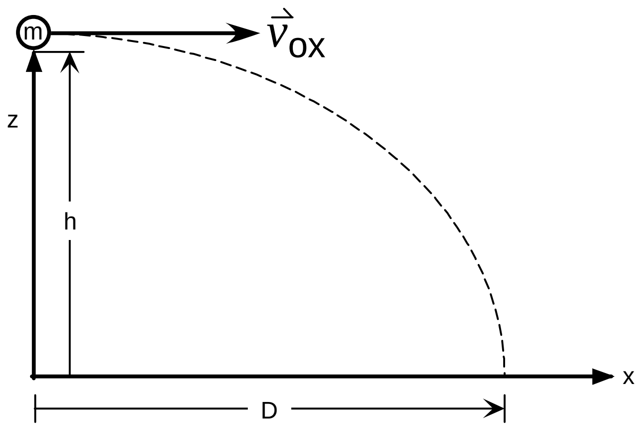
Fall 2008

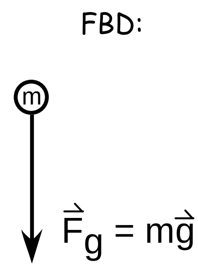
Introduction

The purpose of this experiment is to measure the initial velocity of a projectile, then calculate the distance it will travel when fired at an angle. A wastebasket placed at this distance will demonstrate your understanding of the theory.

Theory

We begin with an FBD for a ball shot horizontally from a table, neglecting air resistance.





Since the force is constant, we can immediately write the following:

$$z = \tfrac{1}{2} a_z t^2 + v_{oz} t + z_o \quad \text{(Eqn. 1)}$$

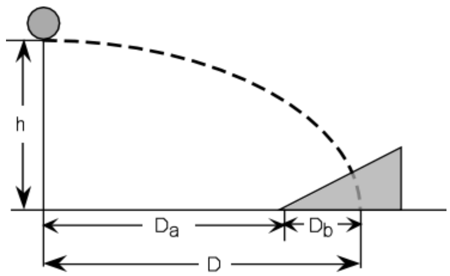
$$x = \tfrac{1}{2} a_x t^2 + v_{ox} t + x_o \quad \text{(Eqn. 2)}$$

From the force diagram, $a_x = 0$, and $a_z = -g$. We can use these two equations to determine how much time it takes for the projectile to hit the floor. Then, from *where* it hits ($x_f = D$), we can determine how fast it must have been going when it left the launcher, v_{ox}.

1. We need values for x_o, z_o, v_{ox}, v_{oz}, a_x, and a_z (the coefficients in the polynomials (1) and (2) above). We can use the symbols in the sketch for the initial positions and velocities, and from the FBD get the accelerations to rewrite these six coefficients for our projectile (three of them are zero!).
2. Now rewrite Eqns. (1) and (2) with the new coefficients and solve them for $t_f\,(g,h)$, the time to hit the floor when $x_f\,(g,h) = D$, the horizontal distance traveled before hitting (it's not necessary to solve for t numerically!).
3. Finally, derive an expression for $v_{ox}\,(g,h,D)$, the *ballistic* muzzle velocity of the ball as a function of g, the height of the ball, and the distance it travels. (Your final expression should contain g, h and D, ***not*** their numerical values!) Check that your derived expression produces a result with units of *velocity*!

Experiment

Part I: Measuring the Muzzle Velocity



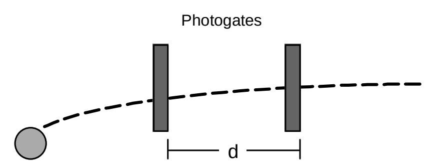
Ballistic Method

1. Your instructor will show you how to load and shoot the spring launcher *horizontally* ($\theta = 0^o$) from a table top. Measure the height, **h**, of the projectile above the floor (measure to the *bottom* of the picture of the projectile on the right side of the launcher).

   Note: Be sure to set the launcher for ***medium*** *range throughout the experiment; the long range setting will hit the ceiling or opposite wall!*

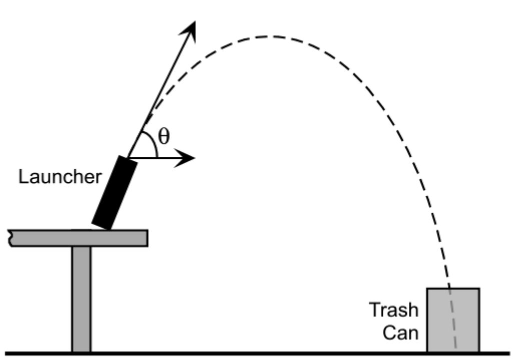
2. Fire the launcher a few times, noting the place the projectile hits the ground. *Be sure to check the angle of the launcher after each shot*. Place a wooden catch box at this location; fire a few more shots to check its position (it's helpful to use a piece of tape to mark the front position of the box; don't remove this tape until you are ready to leave the lab). Measure the distance along the floor from a position directly below the release point of the projectile (the side of the launcher is marked with a small cross) to the front edge of the box. Call this distance $\mathbf{D_a}$.
3. Place a sheet of carbon paper (ink-side up) under a sheet of graph paper, and *lightly* tape the graph paper to the bottom of the catch box, aligned with its front edge. Fire the launcher *at least* 10 times. The projectile will leave a mark on the graph paper; measure the distance from the front edge of the paper to the center of each spot, $\mathbf{D_b}$.
4. Calculate the horizontal distance, $\mathbf{D} = D_a + D_b$ the projectile traveled for each shot. Also calculate the minimum, average, and the maximum distance; denote these as $\mathbf{D_{min}}$, **<D>**, and $\mathbf{D_{max}}$.
5. Use the equation you derived for the ballistic muzzle velocity to find the minimum, average, and maximum velocity of the projectile.

Time of Flight



6. Attach the double–photogate assembly to your launcher. Position it so that the first photogate is as close to the end of the launcher as possible. Set the timer to *pulse, 0.1 ms, memory on*.
7. Measure the muzzle velocity directly using a pair of photogates. Compare the average *ballistic* velocity with the average *time-of-flight* velocity, and report the percent difference. Note: If the photogate velocity is much larger or smaller than v_{max} or v_{min} you should check your measurements and calculations. A 3% difference with the average velocity can be too much!
8. Remove the photogate assembly from the launcher.

Part II: Sink the Ball!



9. The algebra becomes a little (but not much) messier when the ball is shot at an angle, so you'll use a Microsoft Excel spreadsheet to determine the distance your projectile will travel. Set your launcher to any angle between 30^o and 50^o. The height of the projectile should be the same as before (*check it!*).

10. Open the Excel spreadsheet *Projectile Motion* in T:\Phys151 (be sure to click “Enable Macros”), and enter the projectile height, launcher angle, and your velocities. Click the “Graph” button to see the trajectory of the projectile. Add your names to the graph header (following the instructions at the bottom of this page) and print a copy of the graph for each person in your group. Use your graph to decide where to place a wastebasket to catch the projectile when fired (feel free to draw on your graph!). Draw a sketch in your report, make your measurements, and take your shot. The rules are as follows:

    I. The placement of the trash can is accomplished by calculation and measurement, *not* trial and error. Once the can is placed on the ground, it *cannot* be moved closer to, or further from the launcher; only left-right adjustments will be allowed.

    II. If you miss the can on the first attempt, you will be given a second shot, *without* moving the launcher or the can (this will allow for misfires of the launcher). Further attempts will not count. Rim shots count only if the projectile bounces into the can (which rarely happens).

    III. Getting the projectile in the trash can on the first try gains you a bonus of 0.5 points on your lab grade!

    IV. For the more daring amongst you, smaller targets will be made available (as time allows).

It is traditional to gather the whole lab section together to share in your success!

Discussion

- Report your numerical results: the three muzzle velocities, the photogate velocity and the percent difference.
- Make sure that your earlier derivation shows how you connected the six coefficients in Eqns. (1) and (2) to the values in Part I.
- Discuss possible reasons for any difference between the ballistic and time-of-flight velocities.
- Describe how you determined the placement of the trash can.
- If the ball missed the can on the first try, discuss the reasons why it missed.
- Attach the target sheet to the back of the lab report of one member in your group.

Before leaving the lab, please remove any tape you applied to the floor or the lab bench.

Adding Your Names to the Header of the Excel Trajectory Graph:

A. When viewing the graph, click the Microsoft Office Button (on the upper left side), choose “Print”, and then “Print Preview.

B. Click the **Page Setup...** button, then the **Header/Footer** tab. Next click the **Custom Header...** button, and replace “Your Names Here!” with the appropriate entry in the Center Section. The code that appears in the Right Section will automatically insert the current date.

C. Click **OK** twice to return to the Print Preview window, then click the **Print** button. Print a copy for each person in the group.