

Kepler's 2nd Law

Fall 2008

Introduction

In this experiment, you will examine the motion of an air puck traveling in an elliptical path. Applying the theory of conservation of angular momentum, you will see if the puck moves as predicted by Kepler's 2nd Law.

Theory

The experimental apparatus consists of a table with a sheet of carbon paper and newsprint on top. A 550 g air puck is attached to the table by four springs (Fig. 1), and connected to a spark-timer and an air supply. The puck rides on a cushion of air so that there is very little friction between it and the newsprint. The spark-timer records the position of the puck on the newsprint at specific intervals.

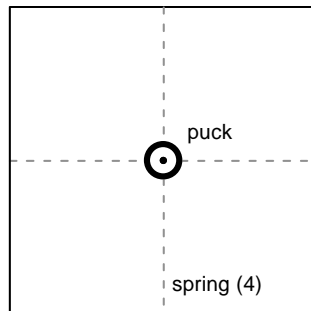


Fig. 1: Puck at rest

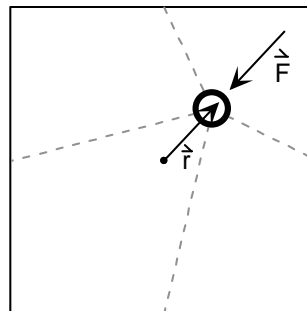


Fig. 2: Puck in motion

When the puck is displaced a distance \vec{r} from its equilibrium position, it experiences a restoring force, \vec{F} created by the springs (Fig. 2). If $\vec{F} \parallel \vec{r}$, then the torque $\vec{\tau} = \vec{r} \times \vec{F} = 0$, and the angular momentum, \vec{L} is constant. Constant angular momentum implies Kepler's 2nd Law: an object traveling in an orbit will sweep out equal areas in equal intervals of time (Fig. 3). For this to be true, the *speed* of the object must change as it travels along its path.

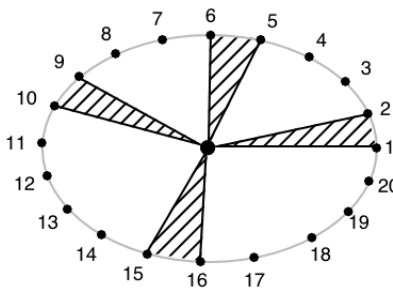


Fig. 3: The shaded areas are equal.

Experiment

1. Turn on the air supply and practice releasing the puck in a circular motion. Keep the size of the orbit about that of a standard letter-size sheet of paper; this will keep your areas more consistent with each other. Be very careful that you do not tilt or push down on the puck, as the carbon paper underneath is very easily torn; holes in the carbon paper will inhibit the puck's frictionless motion. Lightly mark the paper with an arrow indicating the direction of rotation.
2. The spark timer should be set to **StandBy** and the rate to **15 cps** (counts per second). With the puck at rest, have your partner quickly turn the knob from **StandBy** to **Spark On**, and back to **StandBy** again. This will mark the paper at the center of the pucks orbit.

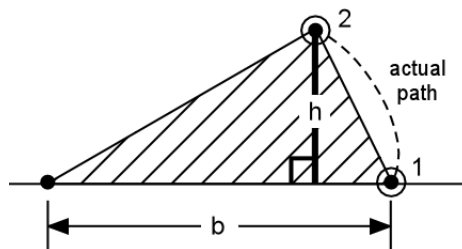
CAUTION: Keep your hands off the edges of the air table and the puck while the spark timer is on. You could get a shock from the apparatus.

3. With the spark *off*, start the puck moving in a circular path. Your partner will again turn on the spark timer, then turn it off when the puck has made one complete rotation.
4. Remove the puck and the sheet of newsprint; the orbital path will be marked on the underside of the sheet. *Indicate the starting and end points, the direction of motion, and the setting of the spark timer on the paper.*

Note: It is very important that $t = 1$ spark is marked as the *first* point!

Analysis

5. Carefully circle and number every point on the newsprint.
6. Use a ruler to draw a line connecting each point to the center of the orbit. Then draw a line connecting *every other pair* of adjacent points (e.g. between point 1 and point 2, then between point 3 and point 4, etc.) as shown in the figure at right.
7. Finally, for the point pairs that were connected, draw a line that starts at one point, and intersects the opposite line *perpendicularly*, as shown in the figure. This line will represent the "height" of each triangle.
8. Measure the base and height for each triangle drawn. Use these measurements to calculate the *area* swept out in a particular time interval, and record in a table (estimate your measurements to 0.1 cm):



Time Interval	Sparks	b (cm)	h (cm)	$A = \frac{1}{2}bh$ (cm ²)
1	1 – 2	—	—	—
2	3 – 4	—	—	—

9. If angular momentum is conserved, then the areas of these triangles should be the same. Use KaleidaGraph to create a plot of area vs. the time interval. What would you expect this graph to look like? Place a linear fit on the graph to see if there's a noticeable trend. (You might want to adjust the scale of the vertical axis if your data is jammed along the top of the graph.)
10. Create another KaleidaGraph plot of the residuals vs. the time interval (see "Graphing & Curve Analysis with KaleidaGraph"). A pattern in the residuals might indicate that a systematic error occurred during the experiment. Do you notice any pattern?

Project (Optional, +1 point):

- Carefully align the transparency grid *along the major axis* of the ellipse, and then find the coordinates of each point. Tabulate your results as shown below (use *gu* – grid units – for the units). (*Note: It's important that you take care when aligning the grid along the major axis!*)

<i>t (sparks)</i>	<i>x (gu)</i>	<i>y (gu)</i>
1	—	—
2	—	—

- Use KaleidaGraph to see if the orbit is *really* an ellipse:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

*Hint: Plot y^2 vs. x^2 . 'a' and 'b' are the semi-major and semi-minor axes, respectively. What *should* this graph look like? *Note: Plotting y vs. x and saying it looks like an ellipse is not a valid demonstration! Note that the 'a' and 'b' values from KaleidaGraph are **not** the semi-major and semi-minor axes!**

Discussion

- Was angular momentum conserved? What did your graph of area vs. the time interval show? If angular momentum was not conserved, explain what you think would account for this deviation from expected behavior?
- By what percentage did the area decrease (from the largest to smallest area)?