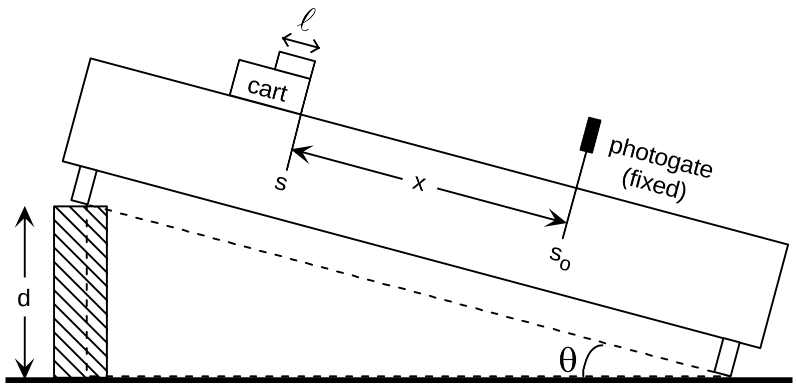
Conservation of Energy on an Inclined Plane

Fall 2008

Purpose

Today we will perform a variation of the experiment described in section C6.3 of Moore. We seek to demonstrate that the final kinetic energy of an object sliding down a frictionless incline from rest is proportional to the vertical distance fallen.

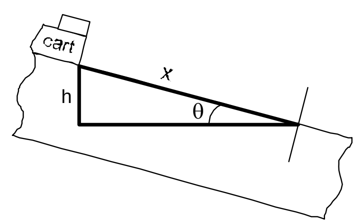


Theory

If the cart were falling *vertically* a distance *h*, as shown below, then it is easy to get the following result by applying energy conservation:

$$\frac{K_f}{m} = gh \qquad \{\text{Eqn. 1}\}$$

In this experiment, the cart is sliding without friction down an incline. But since the interaction between the track and glider is always perpendicular to the cart's velocity, we can ignore this force and the equation above still holds. Since *h* is the vertical distance the cart falls, you can see from the figure at right that $h = x \cdot \sin\theta$.



Substituting *h* into Eqn. 1 gives:

$$\frac{K_f}{m} = gx\sin\theta$$

$$but \quad K_f = \tfrac{1}{2}mv^2$$

$$so \quad \frac{v^2}{2} = g\sin\theta x$$

So, plotting $\frac{v^2}{2}\left(=\frac{K_f}{m}\right)$ vs. *x* should produce a linear graph with slope $g \cdot \sin\theta$ and the y-intercept through the origin.

Procedure

I. *Track set up*

1. The photogate timer needs to be placed in position *before* the air supply is turned on. Hold the cart with its leading edge at a convenient track position, say 160 *cm*. Turn the timer on (set to "gate" mode, 0.1 *ms*, memory *on*), and move it along the track until the red LED on top of the gate lights; this indicates that the timer will be triggered by the leading edge of the flag. This defines the coordinate of s_o. *Note: You'll need to check the position of the photogate timer after the track has been inclined, since the photogates don't move with the track.*
2. Turn the air supply on. Make sure your air track is level by adjusting the screws so that a cart at rest remains so.
3. Tilt the track by placing a block under one end, and *very carefully* measure the quantities you need to calculate θ, the angle of the track elevation from the horizontal. For recording these data, put the numbers in a large, clear sketch of the tilted track, then show your calculation of θ (double check your measurements; this is often a source of error!).
4. Again check the position of the photogate. Tape the timer to the bench so that it will not move during your experiment. Measure the length of the flag using a vernier caliper.

II. *Measurement of* K_f/m *vs. x*

5. Set up a data table with the following headers (you'll be collecting a lot of data, so start at the *top* of a clean sheet of paper):

s_o (cm)	s (cm)	$x = s_o - s$ (cm)	t (s)	$\langle t \rangle$ (s)	v (cm/s)	$\frac{v^2}{2}\left(=\frac{K_f}{m}\right)$ $(\text{cm/s})^2$

6. Move the cart up the track to position s = 20 *cm* which will give the longest distance x. Hold the cart in place with the tip of your pencil eraser; then release it by pulling your pencil straight out. The photogate will measure the time it takes the flag to pass through the timing gate.
7. In order to choose scales for your graph of K_f/m vs. x, measure t from the largest s, and then use the resulting x and K_f/m to scale a graph. Repeat (and record) the measurements with the same x until you are satisfied the data have settled. Draw a light line through time measurements you think are wrong. Calculate the average time, $\langle t \rangle$ and velocity, and plot the point. *It is very important that you plot your points as they are calculated!*
8. Set x to the smallest value possible and measure t. Repeat for other values of x, using your graph to choose values of x to fill the gaps; 8 or 10 points will do. Be sure to spread your points out over the *entire* length of the track.

Analysis

8. Once you have a completed graph, use KaleidaGraph to fit a linear function of the form $y = a + bx$ to your data (be sure to choose the *General* fit, "Linear w/uncertainties"). Follow the instructions in "Graphing & Curve Analysis using KaleidaGraph" to set up a data sheet and run the analysis. Be sure to include the uncertainty with your results (use *twice* the calculated uncertainty).

   You should adjust the KaleidaGraph plot so that the origin of the axes is displayed: from the **Plot** menu, choose **Axis Options**. Change the *Minimum* value to 0, click the 'Y' tab at the top of the dialog box, again set the *Minimum* to 0, and click OK.

9. Spend some time thinking about what you expect to find for the values of the slope and intercept. Do your results agree with your expectations? If not (and there is a strong possibility that you might find a discrepancy), try to figure out the reason. *Hint:* Consider the meaning of the *y*-intercept.

Discussion

- As in previous labs, give a brief description of what you have been measuring, and write down the numerical results of your analysis (including the uncertainty) and a reference to the graph.
- State the expected values for the slope and intercept. Discuss whether or not you think the values calculated using KaleidaGraph could be equal to the values you expect (*i.e. are the theory and data consistent?*), and if not, why not. Notice that it is, in a way, better if they are *not* consistent because then you can display your understanding of the whole experiment.
- Calculate what the value of θ would have to be to make $g \cdot sin\theta$ equal to the slope on your graph. Compare this "ideal" value of θ with your measured value, and comment on the differences and affect on the final results.