

## Torque & Equilibrium

Fall 2008

### Introduction

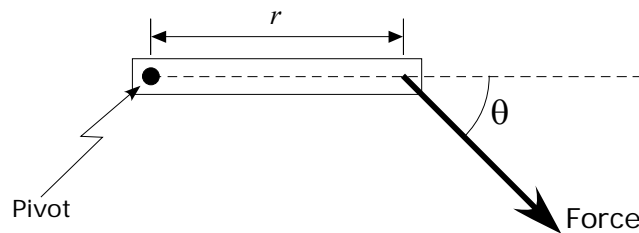
In this experiment you will calculate the torque necessary to keep an *equal arm balance* in equilibrium. You will also balance a rod with a mass on one end and explain its motion by considering the moment of inertia of the system.

### Theory

We have used Newton's Laws to talk about *equilibrium*; equilibrium means that an object is not accelerating because the sum of all the forces acting on the object is zero. In this experiment we introduce the idea of *rotational equilibrium* where an object is not rotating because the sum of the *torques* is zero. Torque can be thought of as a rotational analog of force. The symbol  $\tau$  is used to represent torque:

$$\tau = Fr \sin \theta \quad (1.1)$$

where  $F$  is the applied force; the lever arm,  $r$ , is the distance from the pivot (the axis of rotation) to the point where the force is applied; and  $\theta$  is the angle between  $r$  and  $F$ .

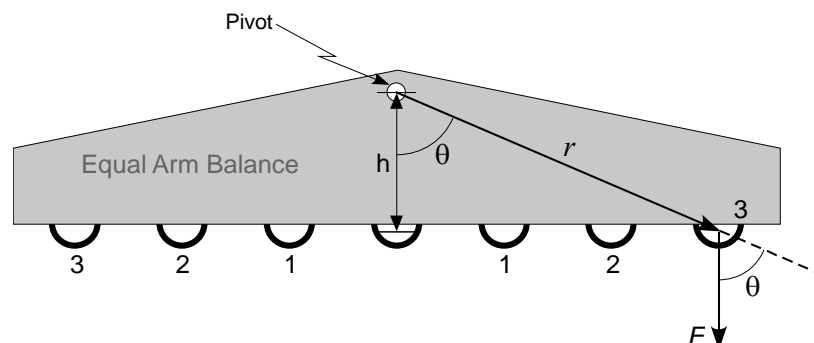


Notice that a longer lever arm results in a larger torque. A larger force also results in a larger torque, as well as a value of  $\theta$  closer to  $90^\circ$ . A torque can cause an object to rotate in a clockwise or counterclockwise fashion. A clockwise rotation is denoted as a negative torque; a counter-clockwise rotation is denoted by a positive torque. For an object to be in equilibrium, the sum of the torques acting on an object must equal zero.

### Experiment

#### Part I: Characterizing the Equal Arm Balance

The equal arm balance has seven loops from which mass can be suspended: three to the left of center, three to the right of center and one in the middle. Since we need the lever arm,  $r$ , and the angle,  $\theta$  for our calculations, we will begin by finding those values for our lever arm, as shown below:



1. Create a data table in your report with the following headers; note that the length measurements are in units of meters:

Table 1

| Hanger position | $h$ (m) | $r$ (m) | $\theta = \cos^{-1}(h/r)$ |
|-----------------|---------|---------|---------------------------|
|-----------------|---------|---------|---------------------------|

2. Use a ruler to measure the values of  $h$  and  $r$  for each hanger position 1, 2, and 3 on the left **AND** right side of the pivot, and calculate the angle  $\theta$ . Note that  $h$  and  $r$  are measured from the center of the pivot to the center of the loop.

## Part II: Torque and Equilibrium

### Overview

You will first get a qualitative feel for the amount of torque necessary to balance the arm. Then you will experimentally determine the mass necessary to balance a torque applied to the balance arm. Next you will calculate the mass required to exert the appropriate torque to balance the arm. Finally, you will compare the measured and calculated mass.

### Check your intuition

3. Suspend a mass of 0.150 kg on the left side of the equal arm balance at position 3. Now use your hand to balance the arm by pulling *straight down* on the right side, first at position 3, then position 2 and finally position 1.
4. Compare the effort required to balance the apparatus at each of the three positions. Which position required that you pull the hardest to balance the arm? Which required the least effort? Briefly explain the difference, in terms of the length of the lever arm.

### Measurement of Mass

5. Create a data table in your report like the one below

Table 2

| Trial | Hanger Position (left side) | $m_{\text{left}}$ (kg) | Hanger Position (right side) | $m_{\text{right}}$ (kg) |
|-------|-----------------------------|------------------------|------------------------------|-------------------------|
| A     | 3                           | 0.150                  | 2                            |                         |
| B     | 3                           | 0.100                  | 1                            |                         |
| C     | 2                           | 0.150                  | 1                            |                         |

6. For each trial, suspend the desired mass ( $m_{\text{left}}$ ) from the *left side position* as indicated in the table, then add mass ( $m_{\text{right}}$ ) to the *right side* hanger position indicated until the system is balanced; record this mass in your table. Remember that the mass hangers are 50 g each.
7. We begin by calculating the torque generated as a result of hanging mass from the *left side* of the balance arm. Create the table below in your report, and fill in the appropriate values of  $r$  and  $\theta$  from Table 1. Calculate the torque on the left side of the balance arm using equation (1.1).

Table 3

| Trial | Left side       |                       |                        |                        | $\tau = m_{\text{left}} g r_{\text{left}} \sin\theta$<br>(N·m) |
|-------|-----------------|-----------------------|------------------------|------------------------|--|
|       | Hanger Position | $r_{\text{left}}$ (m) | $\theta_{\text{left}}$ | $m_{\text{left}}$ (kg) |  |
| A     | 3               |                       |                        | 0.150                  |  |
| B     | 3               |                       |                        | 0.100                  |  |
| C     | 2               |                       |                        | 0.150                  |  |

8. Now we can calculate the torque on the *right* side of the equal arm. Begin by creating the table below; the values of  $r$  and  $\theta$  are again copied from Table 1 (the mass values carry over from Table 2). With all the values in place in the table, use equation (1.1) to calculate the torque on the right side.

Table 4

| Right side |                 |                        |                         |                         |  |
|------------|-----------------|------------------------|-------------------------|-------------------------|--|
| Trial      | Hanger position | $r_{\text{right}}$ (m) | $\theta_{\text{right}}$ | $m_{\text{right}}$ (kg) | $\tau = m_{\text{right}}g r_{\text{right}} \sin\theta$ (N·m) |
| A          | 2               |                        |                         |                         |  |
| B          | 1               |                        |                         |                         |  |
| C          | 1               |                        |                         |                         |  |

9. Compare the torque on the left side to the torque on the right side by finding the percent difference for each of the three trials.

### Part III: Balancing a Rod with Mass on one End

At the front of the lab you will find a rod with two masses on one end. Try to balance the rod with the masses far away from your hand. Next try to balance the rod with the mass near your hand. Which is easier? Explain why using the idea of moment of inertia,  $I$ , where

$$I = \sum_{i=1}^N m_i r_i^2$$

### Discussion

- Create a summary table that lists the torque calculated on the left and right sides, and their respective percent differences.
- What is equilibrium, and how do the left and right torques relate to the condition for equilibrium?

