

## Measuring $g$ with a Simple Pendulum

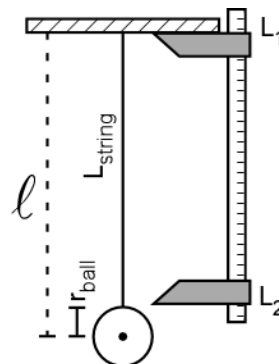
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### Introduction

The length of a pendulum determines its period. You will compare the measured period of a *real* pendulum (spherical ball suspended from a string) with the expected period of an *ideal* pendulum (point mass suspended from a massless string), as determined from Newton's second law. Careful measurements will allow you to determine the acceleration due to gravity,  $g$ , to within 1%.

### Experiment

You are given a pendulum suspended from a support with which you can adjust the length of the string, and a photogate. Use the small cylinder attached to the string to interrupt the timer's light beam. *Set up the photogate so that the beam is interrupted when the pendulum is at rest.* The photogate is set sideways so that the swing angle,  $\theta$ , is kept small (a small angle is assumed during the derivation of the equation found below).



**Pendulum Length ( $\ell$ ):** The analysis *assumes* the length of the pendulum to be the distance from the top clamp to the *center of mass* of the ball. It is easiest to measure the length of the string to the top of the ball, then later add the *radius* of the ball.

**Period ( $T$ ):** In the "PEND" (pendulum) mode the timer will display the time interval for one full swing, the period  $T$  of the pendulum (the light interruption during the back swing is ignored). Make sure the timer is also set to  $1\text{ ms}$ .

- Using a vernier caliper, measure the diameter of the ball along the axis the string passes through. Calculate the radius of the ball.
- Create a data table in your report (*at the top of a fresh sheet of paper*) with the following headings:

$L_2$ (cm)	$L_1$ (cm)	$L_{\text{string}} =  L_2 - L_1 $ (cm)	Period, $T$ (sec)	$\langle T \rangle$ (sec)	$\ell = L_{\text{string}} + r_{\text{ball}}$ (cm)

- For a **string** length about  $90\text{ cm}$  long (the longest you can probably use *on* the table), measure the pendulum's period. Release the pendulum; let it swing once or twice to settle into its natural motion, then press the reset button on the timer to begin collecting data. Repeatedly measure and record the period by pressing the reset button (*don't stop and restart the pendulum!*).
- Calculate the average period,  $\langle T \rangle$ , and the length of the pendulum. Plot this data point on a graph of Average Period as a function of Pendulum Length.
- Shorten the string length, measure the period and calculate the pendulum length. Continue for intermediate lengths down to the smallest you can measure (four or five centimeters). *Plot as you go*, using your graph to "tell" you which length to choose next.

## Analysis

From Newton's second law, the expected period is  $T = 2\pi\sqrt{\frac{\ell}{g}}$ . This expression can be written as  $T = \frac{2\pi}{\sqrt{g}}\sqrt{\ell} = c\sqrt{\ell}$ , where  $c = \frac{2\pi}{\sqrt{g}}$ . Note the units of the constant  $c$  are  $\frac{\text{sec}}{\sqrt{\text{cm}}}$  when  $g = 980 \text{ cm/s}^2$ .

The theory follows a power function:  $y = ax^n$ , where  $n$  is 0.5 (because of the square root). If we use Excel for our analysis, it will attempt to adjust  $n$  as well as  $a$  to get the best possible fit; unfortunately, anything other than  $n = 0.5$  produces lousy results. So, to improve our analysis, we will use a different program, KaleidaGraph, which is better suited to the task of analyzing our data. (*Note: we could still use Excel for our analysis if we linearize the data — i.e. produce a straight-line plot — and plot  $T^2$  vs.  $\ell$* ).

6. Begin creating your table of data in KaleidaGraph by first labeling the data columns (by default, they're labeled A, B, C, etc.). Double-click the column title and change columns A and B to  $L$  and  $\langle T \rangle$  (be sure to include units!!). Then enter your  $\ell$  and  $\langle T \rangle$  data (*Note: Do not include the coordinate  $\{0,0\}$  in your table!*).
7. Now you will create a graph of  $\langle T \rangle$  vs.  $\ell$  as follows:
  - i. From the **Gallery** menu, choose **Linear**, and then **Scatter**. In the "Plot" dialog that appears, click the radio buttons next to  $L$  and  $T$  to graph them on the  $X$  and  $Y$ -axis, respectively (*now you see why it's important to label the columns!*). Click the **New Plot** button.
  - ii. Now you will create the square root function that you need to analyze your pendulum data:
    - a. Select the **Curve Fit** menu, then choose **General**, and then **Edit General...**
    - b. In the dialog box that appears, click the **Add** button, which will add **New Fit** to the list on the left side of the dialog box.
    - c. Select **New Fit** from the list on the left. You'll notice that the name appears in an entry field in the lower left corner of the dialog box. Select this entry field, and change the name to **Square Root**.
    - d. Click the **Edit...** button in the center of the dialog box.
    - e. In the next window that appears, type the following (add a space after the semicolon):
 
$$a*x^{0.5}; a=1$$
    - f. Click **OK** twice to leave the Curve Fit entry menu.
  - iii. Again select the **Curve Fit** menu, then choose **General**, and then **Square Root**. Click the box next to  $\langle T \rangle$ , then click **OK**. A curve will be fit to your data, and a table of results will appear. Click and drag the table to move it on your graph so that it doesn't obscure the curve (*Note: if the table of data does not appear, click the **Plot** menu, and select **Display Equation***).
  - iv. Before printing your graph, you should fix it up a bit.
    - a. Turn off the legend by choosing **Plot, Display Legend**.
    - b. Create a proper graph title (it probably says "Data 1" now); double-click the existing title and edit it. Press Enter, type your names and click **OK**.
    - c. Extend the graph axes so that they include the origin: click the **Plot** menu and choose **Axis Options**. In the dialog box that appears, choose the **X** or **Y** tab, then change the **Minimum** value to 0. Click **OK** when finished.
    - d. Choose **Print Graphics** from the **File** menu to print the graph.
8. Use your measured value of the constant  $c$  to calculate an experimental value for the acceleration due to gravity,  $g$ .

**Discussion**

- Report your measured values of  $c$  and  $g$ .
- Compare the *expected* and *measured* values of  $g$ .
- Discuss how well your data points on the graph fit the theory (the curve drawn by KaleidaGraph). Considering that the theory is derived based upon an *ideal* pendulum, do you think it sufficiently describes the behavior of a *real* pendulum?
- What are some possible sources of error?